teaching costs, fuzzy logic, decision support systems

Małgorzata JUCHA[*], Grzegorz BOCEWICZ[**], Józef MATUSZEK[***]

CALCULATION MODEL OF TEACHING COSTS IN A UNIVERSITY

Abstract

A calculation model of teaching costs in a university is a system of guidelines, notions and relations to facilitate an assessment of the costs generated by individual university departments, majors, subjects etc. The existing calculation models based on the assessment of costs with the use of precision data prove to be ineffective in practice. The major drawback of these systems is the fact that it is not possible to take into account non-precision data in relation to cost generating factors (e.g. the number of didactic groups, hourly rates etc.).

This article presents the author's own proposal of a cost calculation model based on the formalism of fuzzy logics (with the use of the L-R representation). On the basis of the model proposed, it is possible to assess the costs of an academic subject with imprecise information concerning cost generating factors, or the values of those factors are assessed which imply the values set of the cost of a subject.

1. INTRODUCTION

Teaching of students in universities is a process of the provision of educational services. Considering their complex structure, it is not always possible to exactly define some of cost components. There are various methods and ways to determine them, however not all of them are of a practical use [6].

The Institute of Problems of Contemporary Civilization in Warsaw [3] was the first to make an attempt to develop a cost calculation model. This is a complex and inaccurate model. The authors of this model divide costs into three groups: the costs of the existence of a university, teaching costs, costs of studying. The model proposed is described with 22 variables and 22 equations. The complexity level of the computational algorithm is not proportional to the accuracy of the results and the data required today for the purpose of the management of a university. The division proposed provides overall information without any possibility to

[*] M.Sc. Małgorzata Jucha, Koszalin University of Technology, ul. Śniadeckich 2, 75-354 Koszalin, Poland, matiz62@wp.pl

[**] Ph.D. Grzegorz Bocewicz, Department of Computer Science and Management, Koszalin University of Technology, ul. Śniadeckich 2, 75-354 Koszalin, Poland, bocewicz@ie.tu.koszalin.pl

[***] Prof. Józef Matuszek, Department of Production Engineering, University of Bielsko-Biala, ul. Willowa 2, 43-309 Bielsko-Biała, Poland, jmatuszek@ath.bielsko.pl

assess the costs of the tasks performed by the organizational units of the university. In practice, models are necessary which facilitate the valuation of costs of e.g. fields of studies, and this one gives no such results.

The model of teaching costs is built on the basis [6] of two assumptions:

- the didactic hour is the carrier of costs,
- the student is the carrier of the costs of activities.

With the aid of the carrier of costs, the average teaching cost of a student is calculated per one didactic hour. With the aid of the carrier of costs, the number of didactic hours per one student is calculated. The product of these two factors presents the average cost of teaching a student.

The results of calculations in this model aim at the determination of the average cost of providing a major or a subject because of one the type of data is an average hourly rate. Such a method to calculate costs leads to approximations which are too large. This model lacks precision as to the rates applied and the carrier of costs, i.e. the student and the hour. This forces the user to possess information concerning the number of didactic hours in a week or a semester. This model does not take into account the budget which is at the disposal of the university. The calculation of the hours in semesters and in weeks is not justified as the structure of didactic hours is determined at the beginning of the semester, and the number of didactic hours for individual subjects is set in teaching standards.

However, in opposition to the models presented in the literature, the model presented in this study does not deal with the issue of indirect and direct costs, because this is a calculation model of the costs of a subject or a field of study that takes into account the type structure of costs. It enables one to calculate those variables from which the cost depends with a specific budget, and also to assess the budget with given variables. The model proposed provides answers to a number of detailed questions:

- How many didactic groups can be formed with a specific budget allocated to a subject?
- What deviations from the accepted cost level are permissible?
- What cost of a subject can be expected with specific data that forms this cost?
- The proposed calculation model of teaching costs answers these questions.

In accordance with this model, the number of didactic hours is important, which depends from the number of student groups. The advantage of this model is the possibility to use non-precision data, which on the budgeting stage is determined as “between”, “circa”, and “not more than”.

2. CALCULATION OF TEACHING COSTS

2.1. Modeling of teaching costs

In order to implement actions concerning the functioning of a university both in the area of costs and receipts, the managers should be in the possession of an initial calculation, which is prepared quickly and correctly, so that the effects of making a specific decision could be determined.

When preparing to open a new field of study, prior to taking any decision, a university should collect information concerning the following:

- any additional costs to be borne by the university,
- the value to be reached by the costs during the first year,

- the costs that will burden the university's budget in the coming years.

It is impossible to obtain this information with the currently used cost statement, which is based on historic data. This requires a lot of effort and time, which is too a cost value: "time is money".

A solution was proposed based on the results of an analysis of the structure of the expenses by the type conducted on the basis of the report data from several universities [4]. The conclusions from this analysis indicated that it is the staff costs that are the most important costs borne by the university as they constitute over 70% of the total costs. If we exactly assess staff costs, the remaining value can be added as a margin of costs.

For the calculation of staff costs, the data concerning those parameters that have an influence of these costs are required. When analyzing staff costs, it was established that the manager of a unit, before taking a decision concerning opening of a new major, is to know those analytical values which have an influence on the calculation of the final cost, i.e. the following:

- the number of lecture groups,
- the number of exercise groups,
- the number of laboratory groups,
- the staff,
- the remuneration rates of those employed for the needs of the major.

Those in charge of the university are to be familiar with the value of the costs; they should also know what receipts they can expect in connection with the subsidy granted as well as fees for studies related to the opening of a specific major. Owing to this information, it can be determined what the results of decisions taken will be. If a loss results from a specific action, it is to be assessed in what period it will be maintained; a profit is the result, and what its value will be.

In order to obtain complete information in this area, the statement of the teaching costs makes it possible to capture the full teaching cost during one financial year.

In order to find such an arrangement of costs that will provide an answer to the question: how much does one student cost within a year?, one needs to establish and analyze several factors that are required to make managerial decisions, such as the following:

- what product this will be (e.g. a new major),
- data concerning the demand in the area of dynamics,
- qualitative calculations (fashion, demand on the labor market for specific specialists, easy and comfortable studying),
- the picture of the situation on the market.

One needs to pay attention to the fact that concerning setting of fees for educational services, costs are not the only value which limit their amount. Fees for law, medicine and psychology studies may serve as an example. The prices for the abovementioned majors depend from the demand, while the price of extramural studies is not prohibitive to future students. Technical majors are an example where the price for studies constitutes the main factor to undertake studies: they require from the future student a lot of effort during studies. These studies are much more difficult from the point of view of the subjects and the skills which are verified on technical studies during laboratory exercises.

The decision to be taken by the managerial staff of the university concerning undertaking actions aimed at opening a major should depend first of all from the research personnel, secondly, from the laboratories and their equipment, thirdly, from the expenses to be borne in order to obtain a good quality of teaching.

Considering the abovementioned quantitative parameters (e.g. the number of hours, the hourly rate) and qualitative (the teaching level) it is to be stated that the teaching costs are not the only factor on which decisions made in a higher school depend. The cost amounts that are to be calculated as well as the qualitative factors give a complete value of didactic services.

A numerical example is presented below, which demonstrates the significance of the calculation of the basic decision-related factor, i.e. the cost of remuneration.

2.2. Estimation of teaching costs

The purpose of the estimation of costs is to determine which costs are to be taken into consideration when planning to open a new major. In item 2.1, those costs were described which have a substantial impact on decisions related to plans to open a new major. The data concerning the following constitutes the components of staff costs related to conducting didactic class in a major:

the number of didactic hours,

the types of didactic classes,

the hourly rates for those who conduct didactic classes.

$$Ko = \left[(Lh_w \cdot Gr_w \cdot A_w) + (Lh_c \cdot Gr_c \cdot A_c) + \left(Lh_p \cdot Gr_p \cdot A_p\right) + (Lh_l \cdot Gr_l \cdot A_l)\right] \cdot N \qquad (1)$$

where: Ko – the staff cost of the subject,

Lh_w – the number of lecture hours for the subject,

Lh_c – the number of exercise hours for the subject,

Lh_l – the number of laboratory groups for the subject,

Lh_p – the number of project hours for the subject,

Gr_w – the number of lecture groups,

Gr_c – the number of exercise groups,

Gr_l – the number of laboratory groups,

Gr_p – the number of project groups,

A_w, A_c, A_l, A_p – the hourly rate of a teacher who conducts: lectures, exercises, laboratories, projects,

N – the value which increases the costs of remuneration (benefits for employees: 30%), the constant: 1.3.

The personnel cost obtained from dependence (1) is the component of the cost of conducting the subject (2):

$$Kpp = Ko + (Ko \cdot C) \qquad (2)$$

where: Kpp – the cost of conducting a subject,

Ko– the personnel cost (gross remuneration + margins),

C– the proportion of the staff costs to the total costs 2/8 (the costs of maintaining rooms, laboratories, i.e.: energy, materials, external services, depreciation, apparatus).

On the basis of dependence (2), the cost of conducting a subject is calculated when accepting an estimated number of didactic hours. The data concerning the rates of remuneration are calculated on the basis of remuneration tables for those who are academic teachers as specified in the Decree by the Minister of Science and Higher Education concerning the terms and conditions of remuneration for work and granting other benefits related to work for those hired by a public university, dated 22 Dec. 2006 (Journal of Laws No. 251, Item 1852 from the year 2006). The results obtained from dependence (2) are presented in Table 1 on the example of a subject which is conducted on the major of "Information Technology" by the Department of Electronics and Information Technology at the Koszalin University of Technology. This subject is conducted during 44 hours. One professor, who conducts lectures, and an assistant lecturer, who conducts exercises, are involved in the subject. The didactic hours are divided into two types of didactic hours as follows: 22 hours of lectures and 22 hours of exercises. For the purpose of the calculations, it was accepted that the cost of the monthly gross remuneration of the professor will approximately be PLN 5,000, and the cost of the assistant lecturer will be PLN 2,800. The remuneration multiplied by 12 months and divided by the teaching load gives the hourly rate of conducting the didactic classes. The teaching load is the number of the didactic hours conducted by an academic teacher during one academic year, which corresponds to the position occupied in the university in the said example: for the professor, 240 hours of the teaching load was accepted in the calculation of the rate, and 240 hours of the teaching load for the assistant lecturer for the calculation of the rate. The result was multiplied by the number of didactic hours, and then increased by 30 per cent. This increase involves the margins of the remuneration that constitute the costs that the employer has to bear when hiring a staff. These are national insurance contributions, the company's fund of social benefits and the fund of awards.

Tab. 1. Components of the calculation of the costs of conducting a subject for the opening of a new major

Subject	No. of hours	Form of classes	No. of groups **Grw/ Grc**	Lecturer's position	Rate for didactic hour **Aw/Ac**	Costs of remuneration	Staff costs **Kow/Koc**	Costs of conducting a subject **Kpp**
Analogue technology: signals and systems	44	22 lectures	1	professor	PLN 250	5 500	7 150	**28 958**
		22 exercises	4	assistant lecturer	PLN 140	12 320	16 016	

Source: Author's own study

The example presented above demonstrates the dependencies between the cost of remuneration and the data required for its calculation. It also demonstrates the possibility to calculate the total costs of conducting a subject. In order to calculate the cost of conducting a major, the subjects included in the curriculum are to be set; further, the number of hours, the forms of classes and the number of student groups are to be determined. Once these values have been obtained, the personnel is to be assigned to each subject; then, on the basis of a contract concluded with each member of the personnel, the hourly rate of the classes conducted can be determined.

The cost of conducting a major constitutes the total costs of the realization of the subject, which are set in the teaching standards, and are obtained in accordance with the following dependence:

$$Kpk = (Kpp_1 + Kpp_2 + Kpp_3 + \cdots + Kpp_i) \quad (3)$$

where: Kpk – the cost of conducting the major,
Kpp_i – the cost of conducting the i^{th} subject.

What cost will be borne by the university when introducing the subject from the example?

The results of calculations obtained with the use of the model provide an answer to this question and all the other questions set above. They make it possible for those in charge of the university to take a decision concerning the majors that are open or closed.

One needs to remember before taking a decision concerning opening a new major or continuing an existing one that over 70 per cent of the cost of teaching a student involves remuneration and margins.

One also needs to remember it is not only numerical data concerning costs that is required to take managerial decisions. Several factors that are not measurable need to be considered. First of all, the product that is of an interest to us is to be determined, e.g. a new major; the data concerning the demand needs to be collected; a long-term forecast concerning the demand for a given major is to be set, and it needs to be determined how this will change in time.

Those in charge of the university should remember that the cost is a certain consequence of taking decisions concerning majors. For this reason, quick information concerning the cost will give the possibility to provide funds to cover this cost.

The precision data used in the example do not make it possible to determine the teaching costs in a situation when we deal with a plan. It does not include an answer to the following questions asked in universities:

- What will the university enrolment be like? – this constitutes the basis for the planning of finances in the didactic activity in compliance with the dependence as accepted in the example.
- How many student groups will there be?
- How many didactic hours will there be?

In planning, approximate values are to be accepted as it will never be possible to accurately foresee the enrolment numbers, which determines answers to questions: how many groups, how many hours.

For this reason, the model proposed to calculate the teaching costs includes an element of uncertainty in the form of fuzzy numbers.

The model proposed accepts the representation of knowledge in the form of the following pair: a set of decision variables and a set of relations that describe connections between decision variables. The model was formulated in the formalism of the theory of fuzzy sets.

According to dependence (2), budgets can be determined for specific didactic classes or individual majors. Fig. 1 presents the stages of the construction of the university's total costs, whose basic component is the wages budget:

- determining the proportion of the type of conducting of didactic classes for individual subjects,
- determining the costs of the wages budget for the subjects conducted in the unit,
- calculation of the costs for e.g. a department, a unit etc.
- calculation of the costs for a faculty,
- the total costs of the functioning of the university.

The wages fund is the basic component of costs that constitutes ca. 70 per cent of the budget. For this reason, it constitutes the basis for the construction of the budgeting model in the university.

% of lectures conducted,

% of classes conducted,

% of laboratories and projects conducted.

Budgeting of wages fund

Costs of department

Costs of faculty

University's wages fund

Fig.1. Settlement of university's wages fund

The drawback of the model presented in Table 1 is that the data required for the calculation of the cost of conducting a subject is precise, whereas budgeting is based on non-precision data.

A plant involves data that is "circa", "not more than"; these terms are characteristic of fuzzy numbers; for this reason, an analysis of costs and the decision-making process connected with them determines the need to operate on non-precision data. This means that a construction of a system to facilitate a decision support through providing answers to a set of routine questions should be based on a model which takes into account an imprecise nature of the knowledge related to the process described.

3. MODEL OF BUDGETING

In chapter 2, the method was presented to determine teaching costs based on precision data. However, in practice it is required on many occasions to make an estimation of costs without an accurate knowledge of some of the parameters. Therefore, it is necessary to build a model of costs which accepts an imprecise nature of the knowledge possessed. The approach proposed accepts a representation of knowledge in the form of a pair: a set of decision variables (which represent the costs and the parameters connected with them, e.g. Kpp, Ko etc.) and a set of relationships (e.g. dependences of the determination of costs (1), (2), (3) that describe the relationships between decision variables. This model and the problem connected with it is formulated in the formalism of the theory of fuzzy sets.

3.1. Fuzzy model

It was accepted that the model under elaboration includes the following:

- **fuzzy decision variables:**

$$\hat{V} = \{\hat{V}_1, \hat{V}_2, \dots, \hat{V}_n\}$$

where: $\hat{V}$ – a finite set of fuzzy decision variables,
$\hat{V}_i$ – i^{th} fuzzy decision variable.

$\hat{V}_i$ fuzzy variable is a variable that accepts imprecise values represented in the form of fuzzy numbers. A fuzzy number is a set of pairs described in a certain space of discussions X [1]:

$$\{(\mu(v), v)\}, \forall v \in X, \tag{4}$$

where: μ – the membership function of the fuzzy number which assigns to each element $v \in X$ the level of its membership $\mu(v)$ to the fuzzy number, whereas: $\mu(v) \in [0,1]$.

Membership function μ realizes a representation of the space of discussions X of a given variable to range $[0,1]$: $\mu: X \to [0,1]$. The space of discussions is defined on the set of real numbers $X \subset \mathbb{R}$.

In the literature [9], it is usually accepted that the fuzzy number (4) fulfills the following conditions:

1. $sup_{v \in X}\, \mu(v) = 1$, i.e. the fuzzy number (4) is normal,
2. $\mu(\lambda v_1 + (1-\lambda)v_2) \geq \min.\{\mu(v_1), \mu(v_2)\}$, i.e. the fuzzy number (4) is convex,
3. $\mu(v)$ is continuous through intervals.

The abovementioned assumptions apply to fuzzy numbers described in the space of real numbers $X \subset \mathbb{R}$. Further in the paper, it is also numbers described in those spaces that are subsets of natural numbers that will be taken into consideration. In the case of such numbers, the fulfillment of the "convexity" and "normality" conditions is assumed. Fig. 2 presents two fuzzy normal and convex numbers: a number described in the space of real numbers (continuous function $\mu(v)$): Fig. 2 a) and a number described in the space of natural numbers: Fig. 2 b).

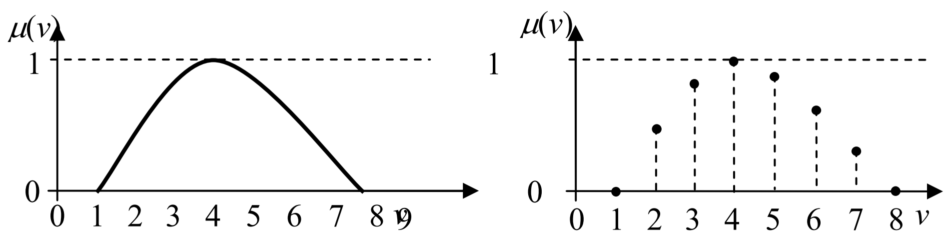

Fig.2. Example fuzzy numbers: a) described in ℝ space, b) described in ℕ space

The number from Fig. 2b is used for the description of values with a discrete nature. For example, if fuzzy number 2b specifies the number of student groups, in the coming academic year we may expect "*circa* 4" groups, not more than 8 and not fewer than 1.

The following was accepted in the context of the discussion above:

$$\hat{V}_i = \{(\mu_i(v), v)\}, \forall v \in X_i, \qquad \hat{V}_i \in \hat{V} \tag{5}$$

which means that fuzzy variable $\hat{V}_i$ accepts an imprecise value that is determined by μ_i and X_i. With the notion of the set of fuzzy decision variables $\hat{V}$, the notion is closely related with the family of the domain of variables $\hat{M}$:

$$\hat{M} = \{\hat{M}_1, \hat{M}_2, \dots, \hat{M}_n\},$$

where: $\hat{M}_i$ – the domain of variable $\hat{V}_i$.
Domain $\hat{M}_i$ is a set of fuzzy values that can be accepted by variable $\hat{V}_i$: $\hat{V}_i \in \hat{M}_i$. It is accepted that $\hat{M}_i$ is defined as follows:

$$\hat{M}_i = \{\{(\mu(v), v)\} : \forall v \in X_i, \forall \mu \in \Theta_i\} \tag{6}$$

where: X_i – the space of discussions being common for all the values of variable $\hat{V}_i$.
Θ_i – a set of the membership functions of the value of variable $\hat{V}_i$.
For example, a set of Gaussian functions with centre (m) from range $[1,10]$ and width (σ) from range [1,2] has the following form:

$$\Theta_i = \left\{ e^{-\left(\frac{v-m}{\sigma}\right)^2} : m \in [1,10], \ \sigma \in [1,2] \right\} \tag{7}$$

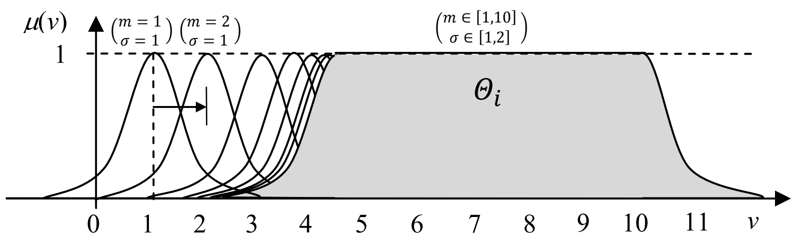

Fig.3. Example set of membership functions

The set of membership functions (4) is presented in Fig. 3. If we accept that Fig. 3 presents the domain of the cost of conducting a subject (in thousand Polish zloty), the fuzzy value of the variable is from the range from "ca. 1 thousand" to "ca. 10 thousand".
In compliance with the above, the domain of each variable $\hat{M}_i$ is determined on the basis of a pair of the space of discussions X_i and the set of membership functions Θ_i, which will be represented by the representation of p:

$$\hat{M}_i = p(X_i, \Theta_i). \tag{8}$$

- **relationships between fuzzy decision variables:**

$$\hat{R} = \{\hat{R}_1, \hat{R}_2, \dots, \hat{R}_m\}, \tag{9}$$

where: $\hat{R}$ – a finite set of relationships between fuzzy decision variables,
$\hat{R}_i$ – i^{th} relationship.

Relationships describe connections between the values of specific fuzzy decision variables. A relationship between two fuzzy variables is the following fuzzy set:

$$\hat{R}_i = \{(\hat{\mu}_i(\hat{v}_a, \hat{v}_b), (\hat{v}_a, \hat{v}_b))\}, \forall \hat{v}_a \in \widehat{M}_a, \forall \hat{v}_b \in \widehat{M}_b, \tag{10}$$

$$\hat{\mu}_i: \widehat{M}_a \times \widehat{M}_b \to [0,1] \tag{11}$$

where: $\hat{v}_a$, $\hat{v}_b$ – the fuzzy numbers that are the elements of domains $\widehat{M}_a$ and $\widehat{M}_b$,
$\widehat{M}_a \times \widehat{M}_b$ – the Cartesian product of domains $\widehat{M}_a$ and $\widehat{M}_b$. Similarly as in the classical definition, the Cartesian product of domains with fuzzy elements is determined as a set of all the ordered pairs of the elements of these sets:

$$\widehat{M}_a \times \widehat{M}_b = \{(\hat{v}_a, \hat{v}_b): \forall \hat{v}_a \in \widehat{M}_a, \forall \hat{v}_b \in \widehat{M}_b\}, \tag{12}$$

$\hat{\mu}_i$ – the membership function of relationship $\hat{R}_i$ with fuzzy arguments. The value of membership function $\hat{\mu}_i(\hat{v}_a, \hat{v}_b)$ is to be interpreted as a degree of the fulfillment of relationship $\hat{R}_i$ for the arguments of $\hat{v}_a, \hat{v}_b$.

An example of relationship $\hat{R}_i$ can be constituted by a relationship that is described with the following statement: "variable $\hat{V}_a$ is smaller than variable $\hat{V}_b$"(e.g. cost $\hat{V}_a$ is smaller than the budget $\hat{V}_b$ possessed). Fig. 4 presents several possible values of variables $\hat{V}_a$, $\hat{V}_b$ and the value assigned to them of the fulfillment of relationship $\hat{R}_i$.

Fig. 4 a) presents a situation when the value of variable $\hat{V}_a$ is certainly smaller than the value of variable $\hat{V}_b$ (the cost is smaller than the budget). The membership functions of the values accepted do not intersect (the areas that are limited with curves do not possess any common parts), hence the value of the membership function of the relationship is 1: $\hat{\mu}_i(\hat{V}_a, \hat{V}_b) = 1$. At the same time, in the situations from Figures 4 b) and 4 c), the membership functions do not intersect any longer, i.e. there are certain values v which belong at the same time to $\hat{V}_a$ and $\hat{V}_b$. In such cases, relationship $\hat{R}_i$is not fulfilled for each value v (it seems that the cost is smaller than the budget: Fig. 4b, and almost certainly the cost is not smaller than the budget: Fig. 4c). In the last figure (Fig. 4d), it can be clearly seen that value $\hat{V}_a$is greater than value $\hat{V}_b$, hence relationship $\hat{R}_i$ is not fulfilled ($\hat{\mu}_i(\hat{V}_a, \hat{V}_b) = 0$) (the cost is certainly not smaller than the budget).

To recapitulate the discussion above: a model can be fulfilled in the form of the following pair: a set of fuzzy decision variables $\hat{V}$ with a family of domains $\widehat{M}$ and a set of relationships $\hat{R}$. In the approach, the model takes on the following form:

$$FM = ((\hat{V}, \widehat{M}), \hat{R}). \tag{13}$$

In the context of the model understood in this way, it is possible to formulate questions related to the search of the value of a specific subset of such decision variables that fulfill a given set of relationships.

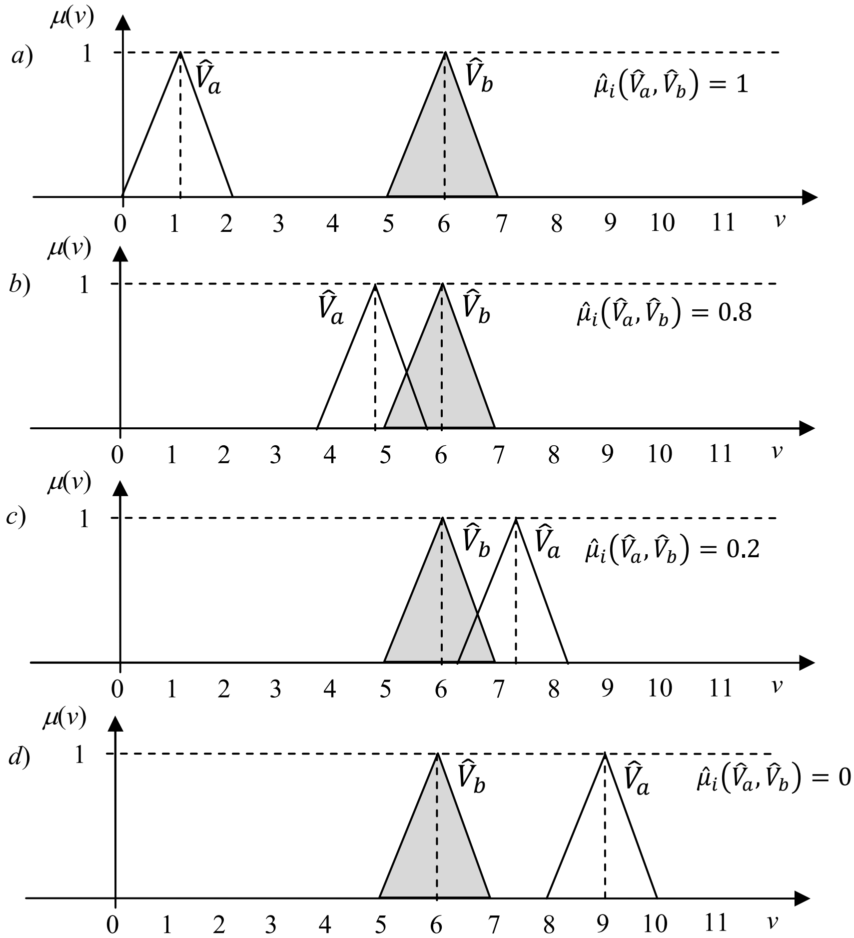

Fig.4. Relationship "variable $\hat{V}_a$ is smaller than variable $\hat{V}_b$": a) $\hat{V}_a$ is smaller than $\hat{V}_b$, b) $\hat{V}_a$ is almost certainly smaller than $\hat{V}_b$, c) $\hat{V}_a$ is almost certainly not smaller than $\hat{V}_b$, d) $\hat{V}_a$ is not smaller than $\hat{V}_b$

3.2. Formulation of the problem

$FM = \left((\hat{V}, \hat{M}), \hat{R}\right)$ is given, which includes the following:

- a set of fuzzy decision variables $\hat{V}$, whose values are determined by $\hat{M}$,
- a set of fuzzy relationships $\hat{R}$ which determines the relationships between variables $\hat{V}$,

In set $\hat{V}$, two subsets of variables were distinguished:

$$\hat{U}, \hat{Y} \in \hat{V}, \quad \hat{U} \cap \hat{Y} = \emptyset.$$

A set of relationships $\hat{R}_{\hat{Y}}$ is given that describe relations between the variables of set $\hat{Y}$. The degree of the fulfillment of relationship S is given that is included in set $\hat{R} \cup \hat{R}_{\hat{Y}}$.

An answer is sought to the following question:

Are there such values of variables $\hat{U}$, for which the relationships included in set $\hat{R} \cup \hat{R}_{\hat{Y}}$ will be fulfilled in a given degree S?

Seeking of such values $\hat{U}$ for which the degree of the fulfillment of the relationships included in set $\hat{R} \cup \hat{R}_{\hat{Y}}$ will be on a given degree S means seeking such values for which membership function $\hat{\mu}_i$ of each relationship of set $\hat{R} \cup \hat{R}_{\hat{Y}}$ takes a value which is at least S.

Distinguishing in the set the decision variables of sets $\hat{U}$ and $\hat{Y}$ means that FM can be perceived as an "input – output" system (Fig. 5).

- "Input" $\hat{U}$ is a set of such variables whose values are not known, and which we want to know.
- „Output" $\hat{Y}$ is a set of such variables which are not known and for which a set of relationships $\hat{R}_{\hat{Y}}$ is given that specifies additional relationships between them. In particular, relationships $\hat{R}_{\hat{Y}}$ can be relationships of an assignment to the variables of specific values. In such circumstances we can say that the values of variables $\hat{Y}$ are known.

In this approach, the problem presented involves seeking such an "input" $\hat{U}$of FM system that will guarantee obtaining "*output*" $\hat{Y}$ that fulfills a given set of relationships $\hat{R}_{\hat{Y}}$. In other words, in the context of a given object, the "*reason*" is sought that guarantees a specific "*result*".

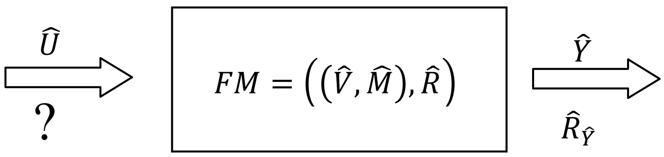

Fig.5. FM as in input/output set

It is to be noted that unlike this type of problems that are dealt with in the literature [8], an imprecise (fuzzy) nature of both decision variables and relationships that connect individual variables is accepted in the problem under consideration. In this way, the solutions sought can "more" or "less" fulfill the assumptions given. This means that a decision maker may on his own determine the limits of the space of solutions through the definition of the degree of the acceptation of individual solutions.

3.3. L-R representation

The problem presented in Chapter 3.2 connected with the FM model (10) is of an overall nature. Depending of the accepted class of decision variables $\hat{V}$ and relationships $\hat{R}$, it can be made more specific and can be used in different decision support areas. In this chapter, a particular case was presented of the FM model that is dedicated to the problem of the calculation of the costs of conducting subjects. The precision consists in accepting $L-R$

representation of fuzzy values and a determination in this representation the form of the relationships corresponding to algebraic operations. In particular, the following is accepted in this model:

- the values of fuzzy decision variables $\hat{V}_i \in \hat{V}$ are described with the aid of the $L-R$ [8] representation. This representation represents fuzzy numbers with the aid of 4 parameters:

$$(m_1, m_2, \alpha, \beta)_{LR}, \tag{14}$$

where: $[m_1, m_2]$ – a range referred to as the core [9] of the fuzzy number, the value of the membership function in this range is 1: $\mu(v) = 1, \forall v \in [m_1, m_2]$

m_1, m_2 – the lower and upper limits of the core,

α, β – the range of the left and right slopes of the fuzzy number.

The four presented (11) determines the parameters of the so-called L and R mapping functions. L and R mapping functions are such that: $L(0) = R(0) = 1$; $L(1) = R(1) = 0$. The membership function of a fuzzy number is defined as follows:

$$\mu(v) = \begin{cases} L\left(\frac{m_1 - v}{\alpha}\right), & \text{when } v < m_1 \\ 1, & \text{when } v \in [m_1, m_2] \\ R\left(\frac{m_2 - v}{\alpha}\right), & \text{when } v > m_1 \end{cases} \tag{15}$$

It was accepted that L and R are linear functions.This means that membership functions accept triangular or generally trapezoidal shapes. Fig. 6 presents an example of a description of a triangular number and a trapezoidal (flat) number.

The assumption of a trapezoidal shape of the membership function for each value of decision variables $\hat{V}_i$ determines the form of the set of membership functions Θ_i. A set of membership functions Θ_i which includes trapezoidal functions only, will further be determined by Π_i.

This means that the domain of a fuzzy variable in L-R mapping is determined similarly to (5) as the following function:

$$\hat{M}_{LR,i} = p(X_i, \Pi_i)\,. \tag{16}$$

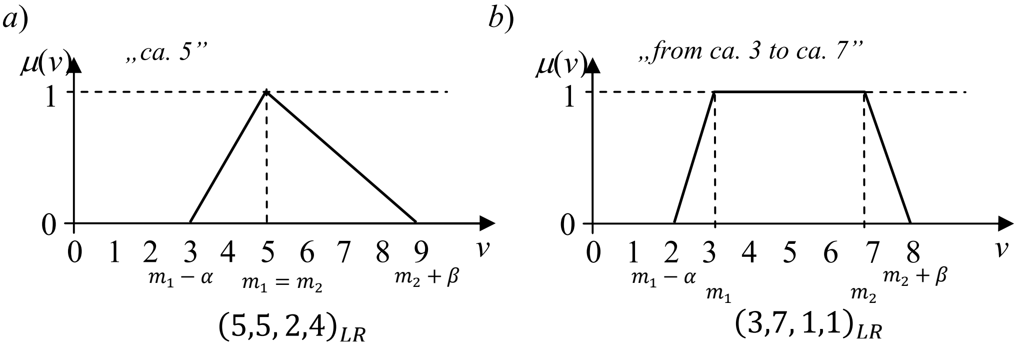

Fig.6. Description of fuzzy numbers in LR mapping, a) “ca. 5”, b) “from ca. 3 to ca. 7”

In particular, $\widehat{M}_{LR,i}$ is a set of fours that represent fuzzy numbers included in the following domain:

$$\widehat{M}_{LR,i} = \{(m_1, m_2, \alpha, \beta)_{LR}: m_1, m_2, \alpha, \beta \in X_i \}\,, \tag{17}$$

where: X_i – the space of discussions of variable $\hat{V}_i$.

In this context, the family of domains for the L-R representation, similarly as $\widehat{M}$, takes on the following form:

$$\widehat{M}_{LR} = \{\widehat{M}_{LR,1}, \widehat{M}_{LR,2}, \dots., \widehat{M}_{LR,n}\}. \tag{18}$$

- a set of relationships $\hat{R}$ includes algebraic relationships only (described with the use of operators: "+", "-", "/", "·", "=", "<", ">", "≤", "≥"). The following expression is an example of such a relationship:

$$\hat{R}_i: \hat{V}_1 + 2{\cdot}\hat{V}_2 = \hat{V}_3. \tag{19}$$

This is an example of an equivalence relation, where the result of the sum $\hat{V}_1 + 2{\cdot}\hat{V}_2$ is compared with the value of variable $\hat{V}_3$. The result of the relation is a set of the values of variables $\hat{V}_1, \hat{V}_2, \hat{V}_3$, for which the membership function of set $\hat{R}_i$ fulfills: $\hat{\mu}_i > S_i$, where: S_i is the acceptation degree of relation $\hat{R}_i$.

Definitions of algebraic operations are required for the construction of this type of relations. For the representation of $L - R$ [8]:

- sum „$\hat{+}$" of two fuzzy numbers ($\hat{V}_1, \hat{V}_2$) is defined as follows:

$$\hat{V}_3 = (\hat{V}_1 \hat{+} \hat{V}_2) = (m_{1,V_3}, m_{2,V_3}, \alpha_{V_3}, \beta_{V_3})_{LR} \tag{20}$$

where: $\hat{V}_1 = (m_{1,V_1}, m_{2,V_1}, \alpha_{V_1}, \beta_{V_1})_{LR}$, $\hat{V}_2 = (m_{1,V_2}, m_{2,V_2}, \alpha_{V_2}, \beta_{V_2})_{LR}$,
$m_{1,V_3} = m_{1,V_1} + m_{1,V_2}$, $m_{2,V_3} = m_{2,V_1} + m_{2,V_2}$,
$\alpha_{V_3} = \alpha_{V_1} + \alpha_{V_2}$, $\beta_{V_3} = \beta_{V_1} + \beta_{V_2}$,

- difference „$\hat{-}$" of two fuzzy numbers ($\hat{V}_1, \hat{V}_2$) is defined as follows:

$$\hat{V}_3 = (\hat{V}_1 \hat{-} \hat{V}_2) = (m_{1,V_3}, m_{2,V_3}, \alpha_{V_3}, \beta_{V_3})_{LR} \tag{21}$$

where: $\hat{V}_1 = (m_{1,V_1}, m_{2,V_1}, \alpha_{V_1}, \beta_{V_1})_{LR}$, $\hat{V}_2 = (m_{1,V_2}, m_{2,V_2}, \alpha_{V_2}, \beta_{V_2})_{LR}$,
$m_{1,V_3} = m_{1,V_1} - m_{1,V_2}$, $m_{2,V_3} = m_{2,V_1} - m_{2,V_2}$,
$\alpha_{V_3} = \alpha_{V_1} + \beta_{V_2}$, $\beta_{V_3} = \alpha_{V_2} + \beta_{V_1}$,

- product „$\hat{\cdot}$" of two positive fuzzy numbers ($\hat{V}_1, \hat{V}_2$) is defined as follows:

$$\hat{V}_3 = (\hat{V}_1 \hat{\cdot} \hat{V}_2) = (m_{1,V_3}, m_{2,V_3}, \alpha_{V_3}, \beta_{V_3})_{LR} \tag{22}$$

where: $\hat{V}_1 = (m_{1,V_1}, m_{2,V_1}, \alpha_{V_1}, \beta_{V_1})_{LR}$, $\hat{V}_2 = (m_{1,V_2}, m_{2,V_2}, \alpha_{V_2}, \beta_{V_2})_{LR}$,
$m_{1,V_3} = m_{1,V_1} \cdot m_{1,V_2}$, $m_{2,V_3} = m_{2,V_1} \cdot m_{2,V_2}$,

$\alpha_{V_3} = m_{1,V_1} \cdot \alpha_{V_2} + m_{1,V_2} \cdot \alpha_{V_1} - \alpha_{V_2} \cdot \alpha_{V_1}$, $\beta_{V_3} = m_{2,V_1} \cdot \beta_{V_2} + m_{2,V_2} \cdot \beta_{V_1} - \beta_{V_2} \cdot \beta_{V_1}$,

- quotient „$\hat{/}$" of two positive fuzzy numbers $(\hat{V}_1, \hat{V}_2)$ is defined as follows:

$$\hat{V}_3 = (\hat{V}_1 \hat{/} \hat{V}_2) = (m_{1,V_3}, m_{2,V_3}, \alpha_{V_3}, \beta_{V_3})_{LR} \tag{23}$$

where: $\hat{V}_1 = (m_{1,V_1}, m_{2,V_1}, \alpha_{V_1}, \beta_{V_1})_{LR}$, $\hat{V}_2 = (m_{1,V_2}, m_{2,V_2}, \alpha_{V_2}, \beta_{V_2})_{LR}$,
$m_{1,V_3} = m_{1,V_1}/m_{1,V_2}$, $m_{2,V_3} = m_{2,V_1}/m_{2,V_2}$,
$\alpha_{V_3} = \dfrac{m_{1,V_1} \cdot \beta_{V_2} + m_{1,V_2} \cdot \alpha_{V_1}}{m_{1,V_2} \cdot (m_{1,V_2} - \beta_{V_2})}, \beta_{V_3} = \dfrac{m_{2,V_1} \cdot \beta\alpha_{V_2} + m_{2,V_2} \cdot \beta_{V_1}}{m_{2,V_2} \cdot (m_{2,V_2} - \alpha_{V_2})}$

Equivalence and minority relations [1] are defined as follows:

- majority relation $\hat{R}_i$: $\hat{V}_1 \hat{=} \hat{V}_2$, whose degree of fulfillment is:

$$\hat{\mu}_i(\hat{V}_1, \hat{V}_2) = \frac{2S^*}{S_1+S_2}, \tag{24}$$

where: S_1 – the size of a fuzzy number that is the value of variable $\hat{V}_1$. The size is calculated as the area of the surface limited by the curve of the membership grade $\hat{\mu}_1(v)$,
S_2 – the size of a fuzzy number that is the value of variable $\hat{V}_2$. The size is calculated as the area of the surface limited by the curve of the membership grade $\hat{\mu}_2(v)$,
S^* – the size of the common part of those fuzzy numbers which constitute values $\hat{V}_1$ and $\hat{V}_2$. Size S^* is calculated as the area of the surface limited by the curves of the membership grade $\hat{\mu}_1(v)$, $\hat{\mu}_2(v)$.

- minority relation $\hat{R}_i$: $\hat{V}_1 \hat{<} \hat{V}_2$; the fulfillment degree is as follows:

$$\hat{\mu}_i(\hat{V}_1, \hat{V}_2) = \frac{{S_1}^L+{S_2}^P}{S_1+S_2}, \tag{25}$$

where: S_1 – the size of the fuzzy number that is the value of variable $\hat{V}_1$,
S_2 – the size of the fuzzy number that is the value of variable $\hat{V}_2$,
${S_1}^L$– the area of the surface limited by the curve of the membership degree $\hat{\mu}_1(v)$ reduced by the area of the surface of common part S^*,
${S_2}^P$– the area of the surface limited by the curve of the membership degree $\hat{\mu}_2(v)$ reduced by the area of the surface of common part S^*.

The set of relationships of an algebraic nature constructed with the use of the operators presented will be marked as follows:

$$\hat{R}_{LR} = \{\hat{R}_{LR,1}, \hat{R}_{LR,2}, \ldots, \hat{R}_{LR,m}\}. \tag{26}$$

In the context of the discussion above, the L-R model takes on the following form:

$$MK = ((\hat{V}, \hat{M}_{LR}), \hat{R}_{LR}) \tag{27}$$

Similarly as in the FM model, for the MK model (21), the following problem is considered: $MK = \left((\hat{V}, \widehat{M}_{LR}), \hat{R}_{LR}\right)$ is given, which includes the following:

- a set of fuzzy decision variables $\hat{V}$, whose values **in the L-R representation** are determined by $\widehat{M}_{LR}$
- a set of fuzzy relationships $\hat{R}_{LR}$ which determines **algebraic relations** between variables $\hat{V}$.

In set $\hat{V}$, two subsets of variables are distinguished:

$$\widehat{U}, \hat{Y} \in \hat{V}, \quad \widehat{U} \cap \hat{Y} = \emptyset.$$

A set of relations $\hat{R}_{LR\hat{Y}}$ is given that describe relationships between the variables of set $\hat{Y}$. The degree of fulfillment S is given of the relationships included in set $\hat{R}_{LR}$ and the degree of fulfillment $S_{\hat{Y}}$ of the relationships included in set $\hat{R}_{LR\hat{Y}}$.

An answer is sought to the following question:

Are there such values of variables $\widehat{U}$ for which relations included in set $\hat{R}_{LR}$ will be fulfilled in a given degree S and relations $\hat{R}_{LR\hat{Y}}$ will be fulfilled in a given degree $S_{\hat{Y}}$?

Unlike the overall problem in this case, the search of the values of variables $\widehat{U}$ is understood to be the search of the values of those parameters (14) that represent a fuzzy number. Seeking parameters and not membership functions alone brings down the problem to the one where variables take on precision values (parameters (14) are understood as variables in this case). Therefore, the problem presented may be brought down to the form of the problem of fulfilling *PSO* constraints, where variables constitute parameters of the membership function of the values of fuzzy variables $\hat{V}$:

$$PS = \left((V_{LR}, D_{LR}), C_{LR}\right), \tag{28}$$

where: V_{LR} – set of variables m_1, m_2, α, β that represent the values of fuzzy variables $\hat{V}$

D_{LR} – a family of the domains of those variables that determine the permissible values of variables m_1, m_2, α, β; it is accepted that the domains are equal to the spaces of discussions X_i.

C_{LR} – a set of those constraints that represent the degree of the fulfillment of relationship $\hat{R}_{LR}$ and $\hat{R}_{LR\hat{Y}}$. The constraints take on the form of the following inequality: $\hat{\mu}_i(V_{LR}) \geq S$ for the relationship of set $\hat{R}_{LR}$ and $\hat{\mu}_i(V_{LR}) \geq S_{\hat{Y}}$ for the relationship of set $\hat{R}_{LR\hat{Y}}$, where $\hat{\mu}_i$ is the function that determines the degree of the fulfillment of relationship $\hat{R}_i$.

Obtaining an answer to the question asked is brought down to the solution of the problem of the fulfillment of constraints (21).

4. CALCULATION EXAMPLES

The examples presented serve to illustrate the use of the MK model to determine the values of the parameters connected with the cost of conducting the subject of Analogue Technology: Signals and Systems. The purpose of the first example is to illustrate the determination of the number of didactic groups with a budget that is known. The purpose of the second example is to illustrate the determination of the cost of conducting the subject of Analogue Technology: Signals and Systems.

4.1. A "backwards" example

The purpose of this example is to illustrate the determination, on the basis of the model proposed, the number of student groups that determines the costs of conducting of a subject that do not exceed a given level of the budget.

The determination of the number of groups concerns the subject of Analogue Technique: Signals and Systems. This subject is foreseen for 44 hours of didactic classes including 22 hours of lectures and 22 hours of exercises. All the factors are known which have an impact on the cost: apart from the number of groups (lecture and exercise groups). The cost of conducting the subject should fit in the budget given amounting to "ca. 30,000 zloty". In this context, an answer is sought to the following question:

Is there such a number of didactic groups which determines obtaining of the cost of conducting the subject of "ca. 30,000 zloty"?

The problem considered has a nature of a "backwards" type. For a given value of the cost, the values of those parameters that guarantee this cost are sought. Due to the uncertainty concerning the real number of newly created student groups after the foreseen enrolment for another academic year, it is required to use a fuzzy model. For this purpose, the MK model (26) was used. In this model, the following set of fuzzy decision variables was accepted $\hat{V}$:

$$\hat{V} = \{\widehat{Ko}_w, \widehat{Ko}_c, \widehat{Ko}, \widehat{Kpp}, \widehat{Lh}_w, \widehat{Lh}_c, \widehat{Gr}_w, \widehat{Gr}_c, \hat{A}_w, \hat{A}_c, \hat{C}_1, \hat{C}_2, \hat{N}_{w1}, \hat{N}_{w2}\}, \tag{29}$$

where: $\widehat{Ko}_w$ – the fuzzy staff cost of the lectures,
$\widehat{Ko}_c$ – the fuzzy staff cost of the exercises,
$\widehat{Ko}$ – the fuzzy staff cost of conducting the subject,
$\widehat{Kpp}$ – the fuzzy cost of conducting the subject,
$\widehat{Lh}_w$ – the fuzzy number of lecture hours for the subject,
$\widehat{Lh}_c$ – the fuzzy number of exercise hours for the subject,
$\widehat{Gr}_c$ – the fuzzy number of exercise groups,
$\widehat{Gr}_w$ – the fuzzy number of lecture groups,
$\hat{A}_w$ – the fuzzy hourly rate of the teacher who conducts the lectures,
$\hat{A}_c$ – the fuzzy hourly rate of the teacher who conducts the classes,
$\hat{C}_1, \hat{C}_2$ – the constant proportionalities of the staff costs to the total costs,
$\hat{N}_{w1}, \hat{N}_{w2}$ – the value which increases the costs of remuneration and margins connected with remuneration.

Variables $\hat{V}$ represent the individual costs as well as the required components for their calculation (in compliance with dependence (1)).

The descriptive relations and the values of costs depend from specific parameters and are formulated in the form of relationship $\hat{R}_{LR}$.

The relations which describe the connections between the value of the cost of conducting the subject $\widehat{Kpp}$ and the remaining variables take on the following form:

$$\hat{R}_{LR} = \{\hat{R}_{LR,1}, \hat{R}_{LR,2}, \hat{R}_{LR,3}, \hat{R}_{LR,4}\}, \tag{30}$$

where:

$$\hat{R}_{LR,1}\colon\ \widehat{Kpp}\cdot\hat{C}_2=\widehat{Ko}\cdot\hat{C}_1 \tag{31}$$
$$\hat{R}_{LR,2}\colon\ \widehat{Ko}=\widehat{Ko}_w+\widehat{Ko}_c \tag{32}$$
$$\hat{R}_{LR,3}\colon\ \widehat{Ko}_c\cdot\hat{N}_{w1}=\widehat{Lh}_c\cdot\widehat{Gr}_c\cdot\hat{A}_c\cdot\hat{N}_{w2} \tag{33}$$
$$\hat{R}_{LR,4}\colon\ \widehat{Ko}_w\cdot\hat{N}_{w1}=\widehat{Lh}_w\cdot\widehat{Gr}_w\cdot\hat{A}_w\cdot\hat{N}_{w2} \tag{34}$$

Relations (31)-(34) constitute a generalized form of dependences (1)-(2) (which takes into account the imprecise nature of the variables) (1)-(2).

In compliance with the assumptions of the MK model, in the set of decision variables $\hat{V}$, input variables $\hat{U}$ and output variables $\hat{Y}$ were distinguished. Those variables that determine the number of student groups $\widehat{Gr}_w, \widehat{Gr}_c$ form a set of input variables $\hat{U}$ while the remaining variables: $\widehat{Kpp}, \widehat{Ko}_w, \widehat{Ko}_c, \widehat{Ko}, \hat{A}_w, \hat{A}_c, \hat{C}_1$, $\hat{C}_2, \hat{N}_{w1}, \hat{N}_{w2}$ form output variables $\hat{Y}$.

For output variables, relationships $\hat{R}_{LR\hat{Y}}$ are known which assign to the variables the values of these variables (margin values, proportionality factors, hourly rates etc.):

$$\hat{R}_{LR\hat{Y},3}\colon \widehat{Lh}_c=(22{,}22,0{,}0)_{LR} \tag{35}$$
$$\hat{R}_{LR\hat{Y},4}\colon \widehat{Lh}_w=(22{,}22,0{,}0)_{LR} \tag{36}$$
$$\hat{R}_{LR\hat{Y},5}\colon \hat{A}_w=(250{,}250,0{,}0)_{LR} \tag{37}$$
$$\hat{R}_{LR\hat{Y},6}\colon \hat{A}_c=(140{,}140,0{,}0)_{LR} \tag{38}$$
$$\hat{R}_{LR\hat{Y},7}\colon \hat{N}_{w1}=(13{,}13,0{,}0)_{LR} \tag{39}$$
$$\hat{R}_{LR\hat{Y},8}\colon \hat{N}_{w2}=(10{,}10,0{,}0)_{LR} \tag{40}$$
$$\hat{R}_{LR\hat{Y},9}\colon \hat{C}_1=(10{,}10,0{,}0)_{LR} \tag{41}$$
$$\hat{R}_{LR\hat{Y},10}\colon \hat{C}_2=(8{,}8,0{,}0)_{LR} \tag{42}$$
$$\hat{R}_{LR\hat{Y},11}\colon \widehat{Kpp}=(30000{,}30000{,}10000{,}10000)_{LR} \tag{43}$$

All the variables with the exception of $\widehat{Kpp}$ accept precision values represented in the form of singletons.

It is to be noted that the relationships that occur both in $\hat{R}_{LR}$ and $\hat{R}_{LR\hat{Y}}$ sets accept the form of "equivalent" relationships whose fulfillment degree $\hat{\mu}_i$ is defined by (24).

In the context of a model defined in this manner, the question concerning the number of didactic groups for the subject of Analogue Technique: Signals and Systems is as follows:

Are there such values of variables $\hat{U}$ (the number of didactic groups $\widehat{Gr}_w, \widehat{Gr}_c$), for which the relationships included in the set and the relationships from set $\hat{R}_{LR\hat{Y}}$ will certainly be fulfilled?

Providing an answer to such a question involves a representation of the MK model as a problem to fulfill PS constraints (28) and solving it with the use of the technologies of programming with constraints (Oz Mozart 11 environment). The set of solutions obtained included only one permissible solution. The number of lecture and exercise groups is as follows:

$$\hat{R}_{LR\hat{Y},1}\colon\ \widehat{Gr}_w=(1{,}1{,}0{,}0)_{LR} \tag{44}$$
$$\hat{R}_{LR\hat{Y},2}\colon \widehat{Gr}_c=(4{,}4{,}1{,}1)_{LR} \tag{45}$$

With $\widehat{Gr}_w$ number of lecture groups "exactly *1*" and of exercise groups $\widehat{Gr}_c$ "*ca. 4*", the cost of conducting the subject ($\widehat{Kpp}$) is between 23953 and 33963 (cf. Fig. 8d).

Owing to the determination of the didactic groups in the form of fuzzy numbers it is possible to establish the range of costs that can be achieved. With the budget given of "ca.

30,000” zloty, 1 lecture group and “ca. 4” exercise groups are to be created. If we create 5 exercise groups, we will exceed the budget by 3,963, and with three exercise groups there are some savings. In general, each cost component $\widehat{Kpp}$ may accept fuzzy values in a specific space of discussions with an assigned degree of certainty.

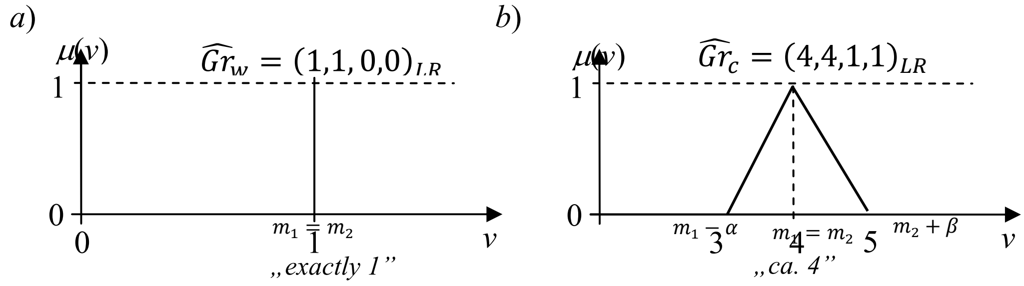

Fig.7. Number of lecture groups $\widehat{\boldsymbol{G}}_{\boldsymbol{rw}}$ a) and exercise groups $\widehat{\boldsymbol{G}}_{\boldsymbol{rc}}$ b) in L-R representation

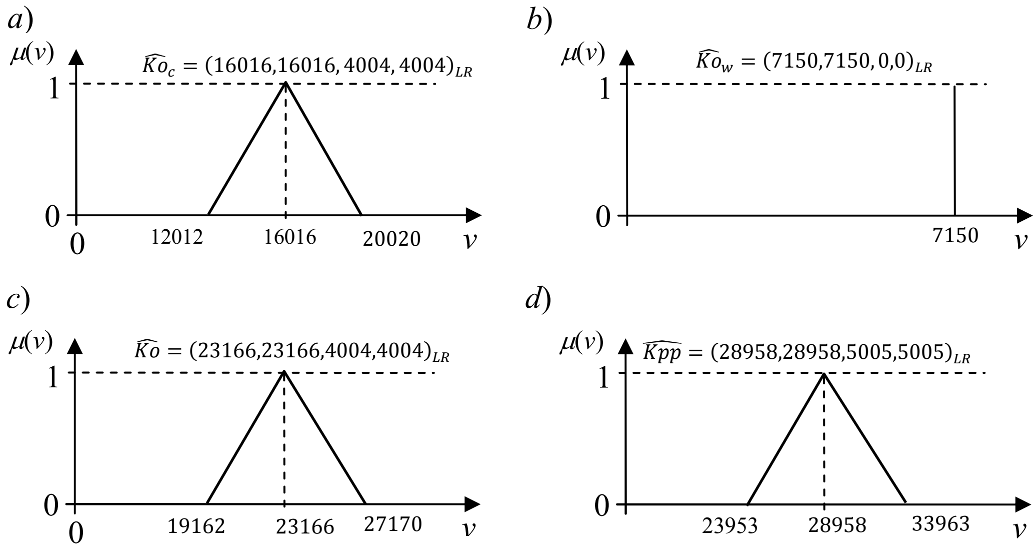

Fig. 8. The costs of conducting the subject: a) the staff cost of the exercises, b) the staff cost of the exercises, c) the staff cost of conducting the subject, d) the total costs of conducting the subject

4.2. “Forwards” example

The purpose of the present example is to illustrate the determination of the value of the cost of conducting the subject of Analogue Technique: Signals and Systems with the fuzzy values of the number of student groups known. The subject is foreseen to involve 44 hours of didactic classes including 22 hours in the form of lectures and 22 hours in the form of exercises. The hourly rate for the lecture is 250, and the hourly rate for exercises is 140. The number of the exercise groups is specified to be "ca. 12", and the number of the lecture groups to be "not more than 2". In this context, an answer is sought to the following question:

What will be the cost of conducting the subject of Analogue Technique: Signals and Systems?

The problem under consideration is of a "forwards" nature. The value of the cost is sought, which is implied by the given values of the parameters.

Similarly as in the "backwards" example, obtaining an answer to the question involves a formulation of the MK fuzzy model of the calculation of costs. In this model, the same set was accepted of decision variables $\hat{V}$ (29) and the set of relationships $\hat{R}_{LR}$ (30).

As opposed to the "backwards" example, the set of input variables $\widehat{U}$ includes cost variable $\widehat{Kpp}$, whereas the set of output variables $\hat{Y}$ includes the following variables: $\widehat{Gr}_w, \widehat{Gr}_c, \hat{A}_w, \hat{A}_c, \hat{C}_1, \hat{C}_2, \hat{N}_{w1}, \hat{N}_{w2}$.

For the output variables, relationships $\hat{R}_{LR\hat{Y}}$ which assign their values to the variables taken on the following form:

$$\hat{R}_{LR\hat{Y},1}: \widehat{Lh}_c = (22,22,0,0)_{LR} \quad (46)$$
$$\hat{R}_{LR\hat{Y},2}: \widehat{Lh}_w = (22,22,0,0)_{LR} \quad (47)$$
$$\hat{R}_{LR\hat{Y},3}: \hat{A}_w = (250,250,0,0)_{LR} \quad (48)$$
$$\hat{R}_{LR\hat{Y},4}: \hat{A}_c = (140,140,0,0)_{LR} \quad (49)$$
$$\hat{R}_{LR\hat{Y},5}: \hat{N}_{w1} = (13,13,0,0)_{LR} \quad (50)$$
$$\hat{R}_{LR\hat{Y},6}: \hat{N}_{w2} = (10,10,0,0)_{LR} \quad (51)$$
$$\hat{R}_{LR\hat{Y},7}: \hat{C}_1 = (25,25,0,0)_{LR} \quad (52)$$
$$\hat{R}_{LR\hat{Y},8}: \hat{C}_2 = (100,100,0,0)_{LR} \quad (53)$$
$$\hat{R}_{LR\hat{Y},9}: \widehat{Gr}_{\mathrm{w}} = (2,2,1,0)_{LR} \quad (54)$$
$$\hat{R}_{LR\hat{Y},10}: \widehat{Gr}_c = (12,12,4,2)_{LR} \quad (55)$$

In the context of the model defined in this manner, the question is as follows:

Are there such values of variables $\widehat{U}$ (the cost of conducting the subject $\widehat{Kpp}$) for which the relationships included in set $\hat{R}_{LR}$ and the relationships from set $\hat{R}_{LR\hat{Y}}$ will certainly be fulfilled?

Providing an answer to this question involves a solution of the problem of the fulfillment of PS constraints that corresponds to the MK model formulated. The solution obtained (the values of cost $\widehat{Kpp}$ and $\widehat{Ko}_w, \widehat{Ko}_c, \widehat{Ko}$) is illustrated in Fig. 9.

For the given values of parameters (46)-(55), the value obtained of the cost of conducting the subject is in the range of 48978-87945 ("ca. 69000"). This is such information that facilitates taking a decision concerning introducing the subject to the curriculum of the studies.

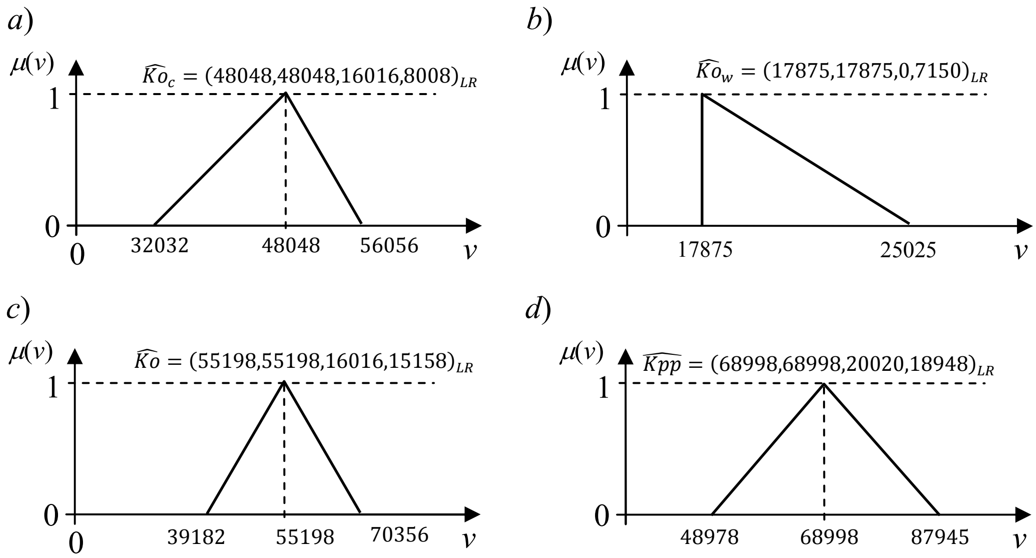

Fig. 9. Costs of conducting the subject: a) the staff cost of the exercises, b) the staff cost of the exercises, c) the staff cost of conducting the subject, d) the total cost of conducting the subject

4.3. Conclusions

The examples presented above prove the fact that with imprecise input data it is possible to calculate the cost of conducting the subject in a specific range. An imprecise nature of input data determines an imprecise nature of the cost of conducting the subject. This means that this model allows one to answer the questions asked with planned data.

The first example demonstrates the application of the calculation model of the teaching cost through backward forecasting. It shows a situation when data is determined with the budget known and a verification is made as to whether this budget will not be exceeded. This gives a possibility to specify those parameters that have an impact on the cost. This is also a mechanism which provides an answer to the following question: are there such parameters that prevent one from exceeding a specific level of costs?

The second example provides an answer as to what value of the costs can be achieved with specific data that has an impact on their amount. This allows one to estimate costs before they are incurred on the basis of imprecise data, which cannot be accurately estimated on the stage of planning.

5. CONCLUSIONS

The calculation model of teaching costs based on fuzzy numbers has filled the gap concerning the lack of strategic information. For this reason, the proposal addressed to the university in the form of a model of a cost statement that is based on imprecise data meets the needs of managers. This model offers a possibility to answer questions concerning the cost of the subject, major, unit and the whole university related to the teaching of students and graduates. It facilitates forecasting with the aid of imprecise data and allows one to introduce historic (precision) data, owing to which it is possible to control the plan with its execution. The deviations occurring serve as an indication and enable one to make the input data more precise as the degree of the imprecision of input data determines the imprecision of output data.

The advantage of this model is the possibility for those in charge of the university to promptly obtain information (through the use of the techniques of programming with constraints) concerning the value of the costs for a specified period of time for the purpose of the valuation of the student cost. In general, it allows one to answer questions concerning the value of costs, and also constitutes the basis for answers concerning each variable from which the total cost depends.

The cost calculation model does not offer a possibility to evaluate an impact of the creation of new majors on the costs of conducting those majors that already exist. The L-R representation with a description of fuzzy variables allows one to solve problems on a scale that is found in practice. Nevertheless, its drawback is that it allows one only to use descriptions of fuzzy variables with the aid of trapezoidal membership functions.

Further research should cover an extension of the model to obtain a greater functionality concerning a comparison of historical data with the forecast obtained through the mapping of the uncertainty factor.

References

[1] BACH I., BOCEWICZ G., BANASZAK Z., MUSZYŃSKI W.: *Knowledge based CP-driven* approach *applied to Multi product small-size production flow*, In.: Control and Cybernetics vol. 39, No.1, 2010, pp 69-65.

[2] BOCEWICZ G., BACH-DĄBROWSKA I., BANASZAK Z.: *Deklaratywne projektowanie systemów komputerowego wspomagania planowania przedsięwzięć*, Akademicka Oficyna Wydawnicza Exit, Warszawa 2009, 302 p.

[3] JANKOWSKA A.: *Koszty kształcenia w szkołach wyższych w Polsce. Model Kalkulacyjny kosztów kształcenia*, Instytut Współczesnej Cywilizacji, Warszawa 1999, 93 p.

[4] KOPCZEWSKI M., JUCHA M.: *Analiza kosztów kształcenia w procesie podejmowania decyzji w szkołach wyższych, Uwarunkowania zastosowań w e-biznesu w gospodarce*. Wyższa Szkoła Ekonomiczno-Informatyczna Warszawa 2008.

[5] LEJA K.: ABC, Forum Akademickie, 2002, nr 3.

[6] MIŁOSZ H.: *Analiza kosztów kształcenia w szkołach wyższych*, PWSZZ Legnica, 2003, 172 p.

[7] NOWAK E.: *Rachunek kosztów*, Ekspert, Wrocław 2000.

[8] PIEGAT A.: *Modelowanie i sterowanie rozmyte*, Akademicka Oficyna Wydawnicza Exit, Warszawa 1998, 678 p.

[9] RUTKOWSKI L.: *Metody i techniki sztucznej inteligencji*, PWN 2005, 436 p.

[10] SASIN W.: *Analiza ekonomiczno-finansowa w praktyce z elementami diagnozy ekonomicznej i rachunku kalkulacyjnego*, Agencja Interfart, Łódź 1994.

[11] SCHULTE CH., SMOLKA G., WURTZ J.: *Finite Domain Constraint Programming in Oz. DFKI OZ documentation series*, German Research Center for Artificial Intelligence, Stuhlsaltzenhausweg 3, D-66123 Saarbrucken, Germany, 1998.

[12] SZUWARZYŃSKI A.: *Identyfikacja podstawowych czynników kosztotwórczych w procesie kształcenia w publicznej szkole wyższej*, Zeszyty Naukowe Politechniki Gdańskiej - Ekonomia, nr 558, Gdańsk 2000, pp. 97-115.