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## MATHEMATICAL MODEL FOR LOGISTIC CONTROL IN INDUSTRIAL CLUSTER

### Abstract

*Entry of the Czech Republic into the European Union (EU) means an expressive expansion of space for European integrated logistics development and application. Basically logistics changes are necessary if the companies and also state in the Czech Republic do not want to be an outsider on the European market. Logistics is one of the important points of enterprise within a cluster. Cooperation of cluster members on the field of logistics for involved companies means a substantial advantage which companies will gain against their competitors not only on the native but also on the foreign markets. Conclusions of the research of mathematical model for logistic control in an industrial cluster are the critical points of this paper. This mathematical model shows the main logistic advantages for cluster members.*

### 1. INTRODUCTION

Logistics is a branch which is largely difficult for mutual and central cluster management. It is a central solution of transportation, supplying and stocking processes.

Cluster must define important activities, critical activities, for a good functioning of companies network. Following graphs (Fig. 1 and Fig. 2) show activities, according to executed research, which clusters consider as the important ones. Graphs, assumed from the study of the European Union, illustrate the importance of logistics for European clusters.

Logistics influences size of minimal supplies by individual cluster members as well. Other **advantages of logistics** for cluster members and for cluster itself are as follows (see Table 1).

From the researches done in the USA in the recent years it is obvious that material handing costs and stocks maintenance are approximately 34 % of total circulation costs, transportation costs 29%, packing costs 12% etc. The most expensive are then handling operations, stocking and transportation. It stands to reason that a slower circulation (above all material for a longer time in stocks and longer material transportation) is a less effectiveness circulation.

Network enterprise creates presumptions for costs reducing and sales increasing of partners in the given business chain.

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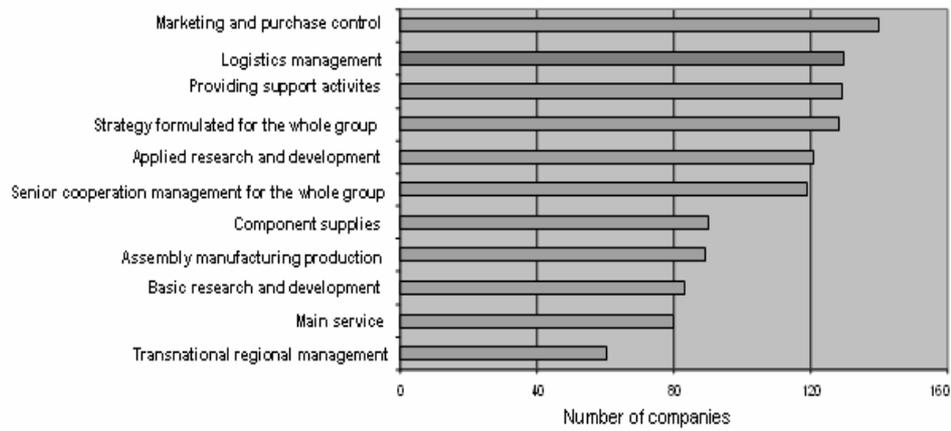


Fig. 1. Importance of activities make for companies in cluster [3]

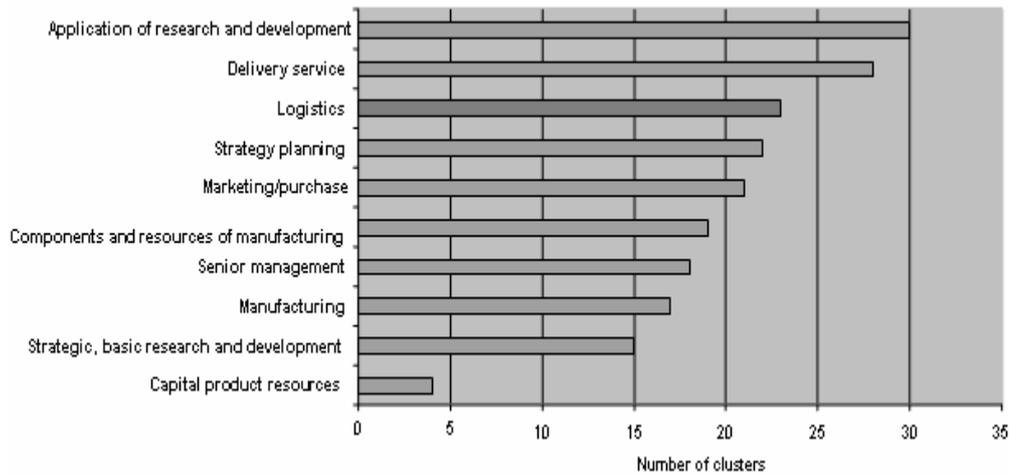


Fig. 2. Main and linked activities on the geographically marked area of cluster – EU [4]

Tab. 1. Advantages of logistics for cluster members and for cluster itself

Activity description	Advantages for company	Advantages for cluster
➤ <i>Corporate stocking</i>	<p>Lower material and product acquisition costs due to corporate central purchase.</p> <p>Lower costs of stock functioning and equipment (one stock for more cluster)</p>	<p>Central optimizing of stock capacities and resources of whole cluster.</p>

<p>➤ <i>Delivery service</i></p>	<p>members).          Opportunity to use stock spaces and resources of central stock.</p> <p>Companies can use distribution cluster net.          Lower material and product transportation costs due to corporate central distribution.</p>	<p>Sophisticated distribution net.          Central management and optimizing of transportation routes among cluster members.          Opportunity to select cluster members on the basis of their product price analysis and material transfer.</p>
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The usage of central stocking is the basic point of “supplying logistics” in the existing chain. We talk about location of central stock in the most convenient place not separately for each cluster member but for the whole cluster. Then agreement with certain suppliers is concluded and these suppliers carry goods not directly in companies but in given central stock. Cluster moves the goods in a central stock and loads the goods in its supplying cars which later hand out goods into the companies.

This structure of goods supply organization from suppliers to companies through a central stock brings many advantages. **Reducing of transportation costs** is one of the advantages which is influenced by reducing of transportation cars importing the goods into one company in cluster. Theoretically speaking always one transportation car which has the goods from a central stock from many suppliers goes to each company.

We give an example: *we have 10 suppliers delivering into a cluster which has 5 cluster members. If we transport the goods from suppliers directly to each company we would need 5 transportation cars from each supplier; that is 50 cars. Each company would have to receive 10 cars. When we use a central stock the number of used cars will reduce to 15 ⇔ 10 cars would go from suppliers to a central cluster stock, material would be moved to the cars for individual customers here and thereby only five cars would go from a central stock to individual companies.*

Goods purchase for **reducing quantity price** is another advantage of stocks centralization in cluster. According to requirements of members cluster will buy goods into a central stock. These goods will be then divided among companies.

**Solution of stocking** in companies of a cluster is another advantage. Thanks to a central stock we talk about reducing of stocks in individual companies of cluster. It is above all the reduction of safety stock in companies. This safety stock is concentrated in a central stock.

Central stocking involves many advantages but disadvantages as well. Central stock functioning costs can be one of the disadvantages. Solution for networks can be transportation without the own central stock. In this case there is a place for using certain carriers, so-called mail-orders that dispose with own costs.

## 2. PROPOSED MATHEMATICAL MODEL

Following part describes a proposed and in Microsoft Office Excel solved model which shows an algorithm of calculations and solutions of some above mentioned areas. These areas are

connected with the problems of supplier selection, storage and logistics control in cluster. We take into account these main areas:

▣ **Supplier selection**

- Calculation of total material costs by individual purchase
- Calculation of total material costs by cluster purchase
- Calculation of total cost saving by the collective cluster purchase

▣ **Localization of central stock**

- Localization of a new central stock
- Company determination in a cluster for central storage

▣ **Stocks control**

- Stocks control of cluster member
- Stocks control in central stock
- Safety stock determination for individual cluster member
- Influence of order change on costs

▣ **Distribution variants determination**

- Evaluation of effectiveness of distribution variants
- Transportation costs and safety stock according to a distribution variant

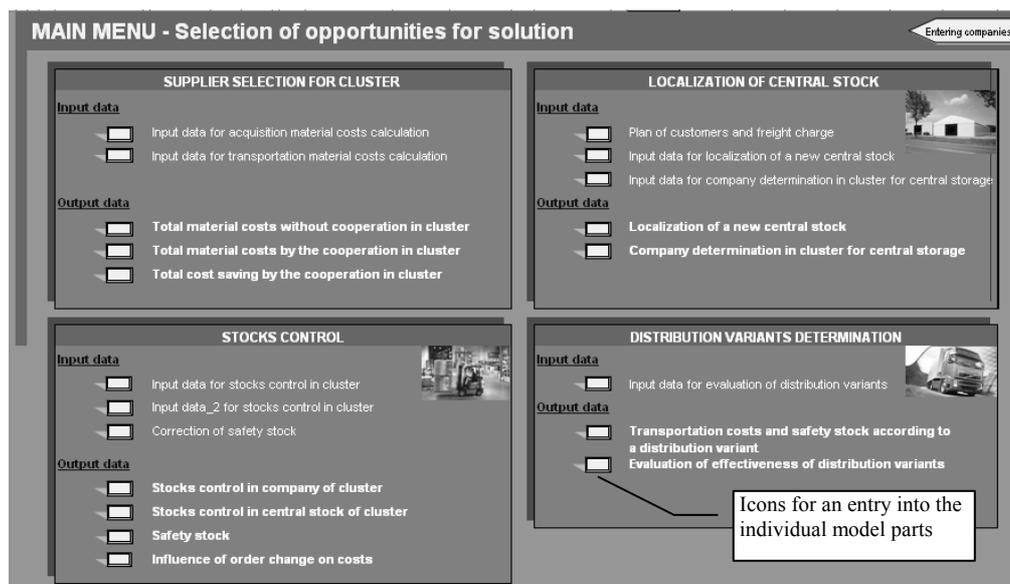


Fig. 3. Selection of opportunities for solution

## 2.1. Mathematical model of supplier selection for cluster

We will consider simplified model that every company gets material directly from supplier.

**Input parameters are as follows:**

- $Q_{kj}$  required quantity of  $k$  material by customer  $j$  ( $g_u$ )
- $p_{kri}$  acquisition price of material corresponding to the group  $Q_{d-1}$ - $Q_d$  acquired  $g_u$  from supplier  $i$  (€)
- $k_{ij}$  transportation distance between supplier  $i$  and customer  $j$  (km)
- $f_{chk}$  freight charge given by supplier (€/km\* $g_u$ )
- $T_{nij}$  transportation input from supplier  $i$  to customer  $j$  ( $g_u$  \*km)
- $T_{nij} = Q_{kj} \cdot k_{ij}$
- $g_u$  general unit (e.g.  $m^3$ , kg, piece)

### A) Calculation of total material costs by individual purchase

We suppose that cooperation among customers on collective purchases does not proceed.

**Sequence of calculations:**

- Calculate acquisition material costs of customer (member)  $j$  at supplier  $i$ .
- Determine transportation input between customer  $j$  and supplier  $i$ .
- Calculate transportation material costs.
- Calculate total material costs as a sum of acquisition material costs and transportation material costs.
- Customer (member)  $j$  will acquire material from supplier  $i$  only in a case when the total material costs will be minimal.

**Solution:**

Acquisition material costs of customer (member)  $j$  at supplier  $i$  ( $C_{aij}$ )

$$C_{aij} = p_{kri} \cdot Q_{kj} \text{ (€)} \quad (1)$$

Tab. 2. Calculation table 1

Member\ Supplier	Supplier 1	Supplier 2	...	Supplier $i$
Member 1	$C_{a11}$	$C_{a21}$	...	$C_{ai1}$
Member 2	$C_{a12}$	$C_{a22}$	...	$C_{ai2}$
...	...	...	...	...
Member $j$	$C_{a1j}$	$C_{a2j}$	...	$C_{aij}$

Transportation material costs  $Q_{kj}$  from supplier  $i$  to customer (member)  $j$  ( $C_{pij}$ )

$$C_{pij} = T_{nij} \cdot f_{chk} = Q_{kj} \cdot k_{ij} \cdot f_{chk} \quad (2)$$

Tab. 3. Calculation table 2

Member\ Supplier	S <sub>m1</sub>	S <sub>m2</sub>	...	S <sub>mi</sub>
M <sub>1</sub>	C <sub>p11</sub>	C <sub>p21</sub>		C <sub>pi1</sub>
M <sub>2</sub>	C <sub>p12</sub>	C <sub>p22</sub>		C <sub>pi2</sub>
....	...	...	...	...
M <sub>j</sub>	C <sub>p1j</sub>	C <sub>p2j</sub>	...	C <sub>pij</sub>

Total material costs of customer (member) j at supplier i (C<sub>tij</sub>)

$$C_{tij} = C_{aij} + C_{pij} \text{ (€)} \quad (3)$$

Tab. 4. Calculation table for selection of linkage between supplier and customer with the minimal costs for material purchase

Member\ Supplier	S <sub>m1</sub>	S <sub>m2</sub>	...	S <sub>mi</sub>
M <sub>1</sub>	C <sub>t11</sub>	C <sub>t21</sub>	...	C <sub>ti1</sub>
M <sub>2</sub>	C <sub>t12</sub>	C <sub>t22</sub>	...	C <sub>ti2</sub>
...	...	...	...	...
M <sub>j</sub>	C <sub>t1j</sub>	C <sub>t2j</sub>	...	C <sub>tij</sub>

**Summary:**

For each customer (member) we choose just that supplier, who will have the minimal costs for material purchase.

**B) Calculation of total material costs by cluster purchase**

We suppose that cooperation among customers on collective purchases takes place. Cost saving happens in discounts when customers acquire a larger quantity of material.

**Sequence of calculations:**

- Calculate total acquisition material costs of customers (1-j) at supplier i. Purchase per price p<sub>kri</sub> corresponding to acquired quantity.  $\sum Q_{kj}$
- Determine transportation input between customer j and supplier i.
- Calculate transportation material costs.
- Customers (1- j) will acquire material from supplier i only in a case when the total material costs will be minimal.

**Solution:**

Acquisition material costs of customer (member) j at supplier i by price p<sub>kri</sub> corresponds to the group of acquired quantity  $\sum Q_{kj}$  (C<sub>acij</sub>)

$$C_{acij} = p_{kj} \cdot Q_{kj} \text{ (€)} \quad (4)$$

Transportation material costs  $Q_{kj}$  from supplier  $i$  to customer (member)  $j$  ( $C_{pij}$ )

$$C_{pij} = T_{nij} \cdot f_{chk} = Q_{kj} \cdot k_{ij} \cdot f_{chk} \text{ (€)} \quad (5)$$

Tab. 5. Calculation table 3

Member\ Supplier	S <sub>m1</sub>	S <sub>m2</sub>	...	S <sub>mi</sub>
M <sub>1</sub>	C <sub>p11</sub>	C <sub>p21</sub>		C <sub>pi1</sub>
M <sub>2</sub>	C <sub>p12</sub>	C <sub>p22</sub>		C <sub>pi2</sub>
...	...	...	...	...
M <sub>j</sub>	C <sub>p1j</sub>	C <sub>p2j</sub>	...	C <sub>pij</sub>

Total material costs of customer (member)  $j$  at supplier  $i$  ( $C_{tcij}$ )

$$C_{tcij} = C_{acij} + C_{pij} \text{ (€)} \quad (6)$$

Total material costs of all customers (members) (1-j) at supplier  $i$  ( $C_{tci}$ )

$$C_{tci} = \sum C_{tcij} \text{ (€)} \quad (7)$$

Tab. 6. Calculation table for total material costs and for selection minimal total material costs of all customers (1-j) at supplier  $i$

Member\ Supplier	S <sub>m1</sub>	S <sub>m2</sub>	...	S <sub>mi</sub>
M <sub>1</sub>	C <sub>tc11</sub>	C <sub>tc21</sub>	...	C <sub>tc1</sub>
M <sub>2</sub>	C <sub>tc12</sub>	C <sub>tc22</sub>	...	C <sub>tc2</sub>
...	...	...	...	...
M <sub>j</sub>	C <sub>tc1j</sub>	C <sub>tc2j</sub>	...	C <sub>tcij</sub>
C <sub>tci</sub>	C <sub>tc1</sub>	C <sub>tc2</sub>	...	C <sub>tcii</sub>

**Summary:**

Supplier with the minimal total material costs  $\min(C_{tci})$  will be chosen for cluster.

**C) Calculation of total cost saving by the collective cluster purchase**

In order that the cooperation in cluster will be profitable this formula must be accepted:

$$\sum \min(C_{tij}) \phi C_{tci} \quad (8)$$

Total cost saving in cluster is defined as:

$$s = \sum \min(C_{tij}) - C_{tci}$$

(€) (9)

Total cost saving (or loss) by individual cluster members (customers) will be calculated as:

$$s_j = \min(C_{ij}) - C_{ij} \quad (\text{€}) \quad (10)$$

## 2.2 Localization of central stock

For location of central stock in cluster we consider two possibilities.

a) Locate the most suitable place in term of quality input of material units into the given companies for **establishment and development of a new central stock**. Indeed in practice we must take in account environmental factors, for example the price of lots, location availability for supply cars, place equipment with communications resources and other.

b) Determine a company in cluster, which is suitable for location of a central stock. We must consider if the given company is convenient for central storage from the view of availability and capacity and storage possibilities.

For the next calculation steps of central stock localization we consider the following input values:

$f_{chk}$	freight charge (€/km* $g_u$ ) (For cluster members $f_{chk}$ is constant)
$S_j$	demand of customer for period T ( $g_u$ )
$j = 1, \dots, n$	number of cluster members
$T$	long-time period (e.g. 1-3 years)

### A) Localization of a new central stock

In this subtask we take over these input values:

$j = 1, \dots, n$	number of cluster members
$x_j$	x-line of j cluster member
$y_j$	y-line of j cluster member
$S_j$	demand of customer for period T ( $g_u$ )
$f_{chk}$	freight charge (€/km* $g_u$ ) (For cluster members $f_{chk}$ is constant)
$w_j = f_{chk} \cdot S_j$	transportation demand for period T
$e$	equivalent; a very small selected value (e.g. 0,001) necessary for a good function of a selected iterative method
$T$	long-time period (e.g. 1-3 years)
$g_u$	general unit (e.g. $m^3$ , kg, piece)

This model enables finding the optimal lines  $[x,y]$  for location of a new central stock in respect of location of given companies in cluster. We go out from recognized lines of given companies  $[x_j,y_j]$  (lines can be considered e.g. in cm and 1cm on the map = 10 km in reality).

Determination of the lines which minimize the **effectiveness function** is a base of this process:

$$\min z = \sum_{j=1}^n w_j \sqrt{(x - x_j)^2 + (y - y_j)^2} \quad (11)$$

We get final optimal lines from the following steps:

**STEP 1:**

We determine input lines which are a good starting point for solution of an optimal object location. For getting these lines we must put partial derivative of function

$$z = \sum_{j=1}^n w_j ((x - x_j)^2 + (y - y_j)^2) \quad (12)$$

according to x a y equal null:

$$\frac{\partial z}{\partial x} = -2 \sum_{j=1}^n w_j (x - x_j) = 0 \quad \Rightarrow \quad x = \frac{\sum_{j=1}^n w_j x_j}{\sum_{j=1}^n w_j} \quad (13)$$

$$\frac{\partial z}{\partial y} = -2 \sum_{j=1}^n w_j (y - y_j) = 0 \quad \Rightarrow \quad y = \frac{\sum_{j=1}^n w_j y_j}{\sum_{j=1}^n w_j} \quad (14)$$

**STEP 2:**

Now we determine partial derivative of function (11) which we put again equal null:

$$\text{For } x \quad \frac{\partial z}{\partial x} = \sum_{j=1}^n w_j \frac{(x - x_j)}{\sqrt{(x - x_j)^2 + (y - y_j)^2}} = 0 \quad (15)$$

After modification

$$x \sum_{j=1}^n \frac{w_j}{\sqrt{(x - x_j)^2 + (y - y_j)^2}} = \sum_{j=1}^n \frac{w_j x_j}{\sqrt{(x - x_j)^2 + (y - y_j)^2}} \quad (16)$$

$$\text{For } y \quad \frac{\partial z}{\partial y} = \sum_{j=1}^n w_j \frac{(y - y_j)}{\sqrt{(x - x_j)^2 + (y - y_j)^2}} = 0 \quad (17)$$

After modification

$$y \sum_{j=1}^n \frac{w_j}{\sqrt{(x-x_j)^2 + (y-y_j)^2}} = \sum_{j=1}^n \frac{w_j y_j}{\sqrt{(x-x_j)^2 + (y-y_j)^2}} \quad (18)$$

For finding final lines it is necessary to use an iterative method. We take the most suitable method, according to the literature [2], for which we must define a **subsidiary function**:

$$f_j(x, y) = \frac{w_j}{\sqrt{(x-x_j)^2 + (y-y_j)^2 + e}} \quad (19)$$

With the following interconnection of equations (16) and (19) and equations (18) and (19) we get equations for the **final lines**:

$$x = \frac{\sum_{j=1}^n x_j f_j(x, y)}{\sum f_j(x, y)} \quad \text{and} \quad (20)$$

$$y = \frac{\sum_{j=1}^n y_j f_j(x, y)}{\sum f_j(x, y)} \quad (21)$$

If the location of a new central stock on these lines will not be convenient from the view of above mentioned environmental factors (e.g. the price of lots, location availability for supply cars, place equipment with communications resources) we take lines from the next most suitable (the smallest) effectiveness function.

## B) Company determination in a cluster for central storage

We consider these input values:

<b>j</b> = 1, ..., n	number of cluster members
<b>S<sub>j</sub></b>	demand by customer for period T (g <sub>u</sub> )
<b>f<sub>chk</sub></b>	freight charge (€/km*g <sub>u</sub> ) (For cluster members f <sub>chk</sub> is constant)
<b>w<sub>j</sub> = f<sub>chk</sub>·S<sub>j</sub></b>	transportation demand for period T
<b>x<sub>j</sub></b>	real distance between the individual companies (km)
<b>T</b>	long-time period (e.g. 1-3 years)
<b>g<sub>u</sub></b>	general unit (e.g. m <sup>3</sup> , kg, piece)

This model enables to find the most suitable company for location of a central stock from the companies group in cluster. Optimal situation occurs in that case when we have the minimal **effectiveness function**:

$$\min z = \sum_{j=1}^n w_j \cdot x_j \quad (22)$$

Product  $w_j \cdot x_j$  represents **transportation input**.

In this case we found the most suitable company for central storage according to the minimal effectiveness function which represents the total smallest transportation input by transportation of material units from a central stock (central company) to other companies in cluster according to conditions.

If this company will not be convenient for central storage from the view of environmental aspects there is the possibility to evaluate and use for central storage a company with the next smallest effectiveness function.

## 2.3 Stocks control

This model endeavours to reach optimum stocks control in industrial cluster. It includes determination quantity order of central store in dependence on cluster members, frequency of supplies, material stock levels, safety stock levels and others.

In this model is thought a mutual connection in companies network (cluster), therefore problem is solving both in aspect of customer and in aspect of selected supplier.

These problems are solved only for distribution of the same one material type.

### 2.3.1 Stocks control of cluster member

Base of this problem is determination of an optimum supply level and frequency of supplies for companies cooperation in cluster.

*Input parameters are as follows:*

For a customer:

$c_j$	ordering costs of customer (member) $j$ on one supplies (€)
$cs_j$	costs of customer (member) $j$ on stocking unit of quantity on period $T$ (€)
$Q_{max_j}$	supply level by supplier $j$ ( $g_u$ )
$Q_j$	optimum order level by customer $j$ ( $g_u$ )
$f_j$	frequency of supplies by customer $j$
$tc_j$	delivery cycle (days)
$s_{dj}$	daily demand of customer $j$ ( $g_u$ )
$xd_j$	signal store level of customer $j$ ( $g_u$ )

Note: *Signal store is a quantity of material units in stock when it is necessary to enter a new order. This value includes a quantity of material units needed on ordering and delivering time of a new supply and quantity of safety (minimal) stock.*

$x_{ave}$	average stock
$Cc_j$	total costs of customer (member) $j$ for a optimum order quantity (€)

For a central stock:

<b>c<sub>j</sub></b>	ordering costs of a central stock on one supply (€)	
<b>cs<sub>c</sub></b>	costs of a central stock on stocking of one unit for period T (€)	
<b>Q<sub>opt<sub>c</sub></sub></b>	optimum order level by a central stock (for cluster) (g <sub>u</sub> )	
<b>tvo<sub>p-c</sub></b>	term of order fulfilment between customer <b>j</b> and a central stock	(days)

Note: *It means how much time a central stock needs to fulfil an order and deliver a supply to the customer (cluster member).*

**f<sub>c</sub>** frequency (number) of the supplies (orders) for a central stock

**Calculation values for cluster members; customer:**

$$\text{Inventory costs } C_s = x_{ave} \cdot T \cdot c_{j_j} \cdot c \quad (23)$$

$$\text{Order costs on period T } C_j = f_j \cdot c_{j_j} = \frac{S_j}{Q_{max}} \cdot c_{j_j} \quad (24)$$

$$\text{Average stock } x_{ave} = \frac{Q_{max_j}}{2} \quad (25)$$

Minimum of total costs is required:

$$\min C = C_s + C_j = \frac{Q_{max}}{2} \cdot T \cdot c_{j_j} \cdot c + \frac{S_j}{Q_{max}} c_{j_j} \quad (26)$$

$$\frac{\partial C}{\partial Q} = 0 \quad \Leftrightarrow \quad \frac{1}{2} T \cdot cs \cdot c - \frac{S_j}{Q^2} c_{j_j} = 0 \quad (27)$$

$$Q_j = \sqrt{\frac{2 \cdot S_j \cdot c_{j_j}}{T \cdot c \cdot cs}} \quad (28)$$

.....

**optimum order level** by customer **j**

Substitution  $Q_{max} = Q_j$  to equation (26) we expression an equation

$$C_{c_j} = \sqrt{2 \cdot S_j \cdot T \cdot c \cdot cs_j \cdot c_{j_j}} \quad (29)$$

**minimum total costs** by customer **j**

It is necessary to determine delivery cycles with a supplier. To get an equation for this delivery cycles we divide period T by optimum number of orders.

$$tc = \frac{T}{(S_j / Q_j)} = \sqrt{\frac{2 \cdot T \cdot c_j}{S_j \cdot cs \cdot c}} \quad (30)$$

### delivery cycles

Order fulfilment  $tv_o > \theta$  practically pays for this equation. Therefore is necessary to determine a lower limit of the signal store  $xd$ .

$$xd = (s_d \cdot tv_{o_{p-c}}) + xp \quad (31)$$

### signal store level

where  $s_d = \frac{S_j}{T}$  is a daily demand.

In case, when  $tv_{o_{p-c}} = tc$ , we have to order next supply in time the last order.

In case, when  $tv_{o_{p-c}} > tc$ , previous process is not effective to use. We determine a new signal store as a salvage value from  $s \cdot \frac{tv_{o_{p-c}}}{Q_j}$  [2].

### 2.3.2 Stocks control in central stock

In this part are an optimum order level and a supplies frequency for the whole cluster determined. There is a problem how to optimise this problem for a central stock (as a main customer) and for a selected supplier.

*Input parameters are as follows:*

For a central stock:

- c<sub>j</sub>** ordering costs of a central stock on one supply (€)
- cs<sub>e</sub>** costs of a central stock on stocking of one unit for period T (€)
- Qopt<sub>e</sub>** optimum order level by a central stock (for cluster) (g<sub>u</sub>)
- f<sub>e</sub>** frequency (number) of the supplies (orders) for a central stock

For a selected supplier:

- c<sub>j<sub>d</sub></sub>** supplier costs on despatch one supply (€)
- cs<sub>d</sub>** supplier costs on stocking of one unit on period T (€)
- Qopt<sub>d</sub>** optimum supply level by a supplier (g<sub>u</sub>)
- tv<sub>o<sub>p-d</sub></sub>** term of order fulfilment between customer j and a supplier (days)

Note: It means how much time a supplier needs to fulfil an order and deliver a supply to the customer (cluster member).

<b>c</b>	price per piece (€)
<b>S<sub>j</sub></b>	demand by customer <b>j</b> for period T (g <sub>u</sub> )
<b>S<sub>c</sub></b>	total demand by all customers for period T (g <sub>u</sub> )
<b>T</b>	long-time period (e.g. 1-3 years) (days)
<b>g<sub>u</sub></b>	general unit (e.g.m <sup>3</sup> , kg, piece)

**Values calculation for a central store; cluster:**

We can calculate the total required quantity of material by a central stock as a central customer. As we mentioned above there is a mutual cooperation between a supplier and a customer (central stock), generation of a cluster. In this term it is necessary to take into account the advantages of cooperation for obtaining the total minimal costs both for a supplier and for a customer (a cluster).

**Total costs for a cluster (for a central stock)**

$$Cc_k = Cs_k + Cj_k \quad (32)$$

$Cs_k$  ... Stocking costs

$$Cs_k = Cs_d + Cs_c \quad (33)$$

$Cj_k$  ... Ordering and supplying costs

$$Cj_k = Cj_d + Cj_c \quad (34)$$

**Optimum order level  $Qopt_k$  by a cluster (central stock):**

$$Cs_k = \frac{Qopt_d}{2} \cdot cs_d \cdot T + \frac{Qopt_c}{2} \cdot cs_c \cdot T \quad (35)$$

$$Cj_k = \frac{S_c}{Qopt_d} \cdot cj_d + \frac{S_c}{Qopt_c} \cdot cj_c \quad (36)$$

By the conditions

$$Qopt_d = Qopt_c = Qopt_k \quad (37)$$

$T = 1$  (one time unit)  
then we get the equation:

$$\begin{aligned}
Cc_k &= \frac{Qopt_k}{2} \cdot cs_d + \frac{Qopt_k}{2} \cdot cs_c + \frac{S_c}{Qopt_k} \cdot cj_d + \frac{S_c}{Qopt_k} \cdot cj_c \\
&= \frac{Qopt_k}{2} \cdot (cs_d + cs_c) + \frac{S_c}{Qopt_k} \cdot (cj_d + cj_c)
\end{aligned} \tag{38}$$

$$\frac{\partial Cc_k}{\partial Qopt_k} = 0$$

$$\frac{1}{2} \cdot (cs_d + cs_c) - \frac{S_c}{Qopt_k^2} \cdot (cj_d + cj_c) = 0 \tag{39}$$

$$Qopt_k = \sqrt{\frac{2 \cdot S_c \cdot (cj_d + cj_c)}{cs_d + cs_c}} \tag{40}$$

Optimum supplies frequency  $f_k$  for a cluster (central stock):

$$Cs_k = \frac{S_c}{2 \cdot f_d} \cdot cs_d \cdot T + \frac{S_c}{2 \cdot f_c} \cdot cs_c \cdot T \tag{41}$$

$$Cj_k = f_d \cdot cj_d + f_c \cdot cj_c \tag{42}$$

By the conditions

$$f_d = f_c = f_k \tag{43}$$

$T = 1$  (unit time unit)

we get the equation:

$$\begin{aligned}
Cc_k &= \frac{S_c}{2 \cdot f_k} \cdot cs_d + \frac{S_c}{2 \cdot f_k} \cdot cs_c + f_k \cdot cj_d + f_k \cdot cj_c \\
&= \frac{1}{2 \cdot f_k} \cdot (S_c \cdot cs_d + S_c \cdot cs_c) + f_k \cdot (cj_d + cj_c)
\end{aligned} \tag{44}$$

$$\frac{\partial Cc_k}{\partial f_k} = 0$$

$$-\frac{1}{2 \cdot f^2} \cdot (S_C \cdot cs_d + S_C \cdot cs_c) + (cj_d + cj_c) = 0 \quad (45)$$

$$f_k = \sqrt{\frac{S_C \cdot (cs_d + cs_c)}{2 \cdot (cj_d + cj_c)}} \quad (46)$$

The main customer (central stock) is here under consideration as the critical member. According to requirements of a central stock we determine optimal supply levels and frequency supplies depending on requirements of individual cluster members. This solution is optimal both for a supplier and for a customer (for a cluster).

### 2.3.3 Safety stocks

This part of the proposed model solves a problem of safety stock level (minimum stock level), which is necessary to maintain in individual companies (members) of cluster and in a central stock. It is necessary in case of problems with a supply period or a production.

Safety stock level is a very sensitive part of business. If this stock is uselessly high there are a lot of financial resources in. But if this stock level is low there could be a problem with material shortage for a production in case of problems with a supply period or a production. In this case it can cause production delays and financial losses as well.

User can compare in this model safety stock level according to exploited distribution variant - with usage of a central stock, or without a central stocking (see 2.4).

This method of determination of safety (minimal) stock level is based on a term order fulfilment and material usage of individual cluster members.

#### Calculation for individual cluster members

##### A) Without using a central stock

For determination of safety stock level (minimum stock level) for cluster members (companies) we calculate with the term of order fulfilment between customer j and a supplier tvo<sub>p-d</sub> (tvo<sub>h-p-d</sub>) and material usage of an individual cluster member. In this equation we calculate with a number of working shifts used in individual companies (members) as well.

*Input parameters are as follows:*

<b>x<sub>p</sub></b>	insurance stock level (minimum stock levels) (g <sub>u</sub> )
<b>h<sub>x<sub>p</sub></sub></b>	number of hours for keeping safety stock level (hours)
<b>ko<sub>x<sub>p</sub></sub></b>	input value for correction of safety stock level of members (g <sub>u</sub> )

Note: Value ko<sub>x<sub>p</sub></sub> is selected base on experience of an individual company with a supplier or for increasing of safety stock, e.g. stone reserve.

<b>tvo<sub>p-d</sub></b>	term of order fulfilment between customer j and a supplier (days)
<b>tvo<sub>h-p-d</sub></b>	term of order fulfilment between customer j and a supplier (hours);

$$tvoh_{p-d} = \frac{tvo_{p-d}}{24}$$

**Ps** number of working shifts for one day  
**h<sub>ps</sub>** number of working hours for one day (hours)

*For single-shift operation* →  $h_{ps} = 8\text{hours}$

*For two-shift operation* →  $h_{ps} = 16\text{hours}$

*For three-shift operation* →  $h_{ps} = 24\text{hours}$

**s<sub>dj</sub>** daily required quantity of material by company (member) **j** (g<sub>u</sub>)

**s<sub>hj</sub>** hourly required quantity of material by company (member) **j** (g<sub>u</sub>/hours)

$$S_{hj} = \frac{S_{dj}}{h_{ps}}$$

**sp<sub>h</sub>** average hourly required quantity of material by all companies (members) (g<sub>u</sub>/hours);

$$sp_h = \frac{\sum S_{hj}}{j}$$

**ks<sub>i</sub>** shift working coefficient:

*For single-shift operation* →  $ks_1 = 1/3$

*For two-shift operation* →  $ks_2 = 2/3$

*For three-shift operation* →  $ks_3 = 3/3$

Calculation:

$h_{xp} = tvoh_{p-d} \cdot ks_i$  (hours) ... number of hours for keeping safety stock level

$$xp = (h_{xp} \cdot s_{hj}) + ko_{xp} \quad (g_u) \quad (47)$$

**Safety stock level (minimum stock level) for members**

### **B) With using a central stock**

In this calculation variant becomes a change in a calculate value  $h_{xp}$ , when we substitute a value **tvo<sub>p-d</sub>** (**tvo<sub>h-p-d</sub>**) by term of order fulfilment between customer j and a central stock tvo<sub>p-c</sub> (**tvo<sub>h-p-c</sub>**).

Calculation:

$h_{xp} = tvoh_{p-c} \cdot ks_i$  (hours) ... number of hours for keeping safety stock level

Another change in calculation by this distribution variant is that a value for correction of safety stock level  $ko_{xp}$  is not maintained in stocks of individual cluster members but it is „transferred“ to a central stock.

$$xp = h_{xp} \cdot s_{hj} \quad (g_u) \quad (48)$$

**Safety stock level (minimum stock level) for members**

**Calculation for a cluster central stock**

***Input parameters are as follows:***

**$tvo_{c-d}$**  term of order fulfilment between central stock and a selected supplier (days)

Note: *It means how much time a selected supplier needs to fulfil an order and deliver a supply to the customer (central stock).*

**$tvo_{h_{c-d}}$**  term of order fulfilment between central stock and a selected supplier (hours)

$$tvo_{h_{c-d}} = \frac{tvo_{c-d}}{24}$$

**$ko_{xp}$**  input value for correction of safety stock level of members ( $g_u$ )

Note: *Value  $ko_{xp}$  is selected base on experience of an individual company with a supplier or for increasing of safety stock, e.g. stone reserve.*

**$sp_h$**  average hourly required quantity of material by all companies (members) ( $g_u$ /hours);

$$sp_h = \frac{\sum_j S_{hj}}{j}$$

Safety stock level (minimum stock level) of a central stock is calculated for the variant with a central stock. Safety stock level is determined on the term of order fulfilment  $tvo_{c-d}$  and value for correction of safety stock level  $ko_{xp}$ , which are transferred from companies (members) to a central stock. This value is subsequently reduced to 1/3.

Calculation:

$$xp = (tvo_{h_{c-d}} \cdot sp_h) + \frac{1}{3} \sum ko_{xp} \quad (g_u) \quad (49)$$

## Safety stock level (minimum stock level) for a central stock

### 2.4 Distribution variants determination

In this part of the model effectiveness of a calculated distribution variant is compared and evaluated:

#### A) Without using a central stock

In this distribution variant it is not calculated with a central stock. Material distribution is realized from a supplier directly to individual customers, cluster members.

#### B) With using a central stock

In this distribution variant is calculated with a central stock as a „halfway house“ between a selected supplier and individual customers, cluster members.

Accounting basis for the comparing and resulting evaluation of these variants are calculated minimum distribution costs and calculated the fixed costs on insurance stock.

#### *Input parameters are as follows:*

<b>c</b>	price per piece (€)
<b>rent</b>	profitability ratios
<b>cs<sub>j</sub></b>	costs of customer (member) <b>j</b> on stocking unit of quantity on period T (€)
<b>S<sub>j</sub></b>	material demand by customer <b>j</b> for period T (g <sub>u</sub> )
<b>x<sub>p</sub></b>	safety stock level (minimum stock level) according to distribution variant (g <sub>u</sub> )
<b>x<sub>p,kē</sub></b>	fixed costs on calculated safety stock level (€)
<b>ct<sub>j</sub></b>	transportation costs to an individual customer (members) <b>j</b> for period T (€)
<b>ct<sub>c</sub></b>	transportation costs to a cluster central stock for period T (€)
<b>x<sub>p-d</sub></b>	transportation distance between customer <b>j</b> and a supplier (km)
<b>x<sub>p-c</sub></b>	transportation distance between customer <b>j</b> and a central stock (km)
<b>x<sub>c-d</sub></b>	transportation distance between a supplier and a central stock (km)
<b>cp<sub>d-p</sub></b>	freight charge for transportation between a supplier and customer <b>j</b> (€/g <sub>u</sub> *km)
<b>cp<sub>d-c</sub></b>	freight charge for transportation between a supplier and a central stock (€/g <sub>u</sub> *km)
<b>cp<sub>c-p</sub></b>	freight charge for transportation between a central stock and customer <b>j</b> (€/g <sub>u</sub> *km)

Calculation:

#### A) Without using a central stock

$$xp_{kē} = xp \cdot c \cdot cs_j \quad (€) \quad (50)$$

fixed costs on calculated safety stock level

$$ct_j = S_j \cdot x_{p-d} \cdot cp_{d-p} \quad (€) \quad (51)$$

transportation costs to an individual customer (members) **j** for period T

#### B) With using a central stock

$$xp_{k\check{c}} = xp \cdot c \cdot cs_j \quad (\text{€}) \quad (52)$$

fixed costs on calculated safety stock level

$$ct_j = Sj \cdot x_{p-c} \cdot cp_{c-p} \quad (\text{€}) \quad (53)$$

transportation costs to an individual customer (members)  $j$  for period T

$$ct_c = \sum Sj \cdot x_{c-d} \cdot cp_{d-c} \quad (\text{€}) \quad (54)$$

transportation costs to a cluster central stock for period T

Effectiveness of distribution variants is determined by values of minimizing function  $f$ :

$$\min f = (\sum xp_{k\check{c}} \cdot rent) + (\sum ct_j + ct_c) \quad (55)$$

### 3. PRACTICAL APPLICATION OF A PROPOSED MODEL

This model, with some modifications (according to the specific requirements of customer), was used within project in company **ČEZ Měření s.r.o.** (Ltd.)

There are logistics solutions of gauges for five regions in the Czech Republic, exactly for 57 customer places (customer centres). According to a regional map of given areas this problem was solved individually for regions – West, Centre, North, East, Moravia.



Fig. 4. Regions and centres in solution

Every region has few centres which must be supplied. Model described in this article can be (with some modifications) used because solutions of individual regions can represent a structure of cooperative companies in network which use the mutual supplier (in this case Skuteč).

The basic problem of this project is a way of gauges distribution from a central stock in Skuteč (*a possible supplier in network*) to the individual customer places. Customer places are in this model solution certain regional central stocks (*a possible central stock in companies network*) and all centres (*possible cooperative companies in network*) which are able to quantify a plan of their consumption. This project determines the ways of instruments distribution and defines necessary minimal stocks in order to ensure needed quantity of required goods and instruments quantity in the time.

**Stocks minimization** in combination with **amount of transportation costs** was used there as evaluative criterion.

#### 4. CONCLUSION

This research paper shows that handling operations, storage and transportation represent the maximal costs in the logistics area. These material flow costs can be reduce when companies enter in a cluster and cooperate with each other.

Material purchase for reducing quantity discount for cluster members is one of the main advantages of cluster building for companies from the view of logistics. Reducing material flow costs among individual participants, suppliers and customers represents another advantage. We talk about reduction of transportation costs (see above mentioned Supplier selection). Another costs reduction is in financial resources – stocks. The central storage is used for cooperative companies. These advantages lead to the biggest one - increasing of competitiveness and maintenance on present global market above all for small and medium-sized companies.

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