Marcin TOMCZYK*, Barbara BOROWIK**, Bohdan BOROWIK***

IDENTIFICATION OF THE MASS INERTIA MOMENT IN AN ELECTROMECHANICAL SYSTEM BASED ON WAVELET–NEURAL METHOD

Abstract
This paper presents the results of testing of a complex electromechanical system model. These results have been obtained for accepted in simulations the method of identifying an inertia moment of reduced masses on shaft of induction motor drive during the changes of a backlash zone width. The effectiveness of correct diagnostic conclusions enables coefficients analysis of testing signals wavelet expansion as well as weights of a supervised learning neural network. The earlier fault detection of five important state variables, which describe physical quantities of chosen complex electromechanical system has been verified for its correctness during the backlash zone width monitoring in the early stage of its gradual rise. The proposed here algorithm with mass inertia moment changes has proved to be an effective diagnostic method in the area of system changeable dynamic conditions and this has been shown in the resulting changes of backlash zone width.

1. INTRODUCTION

Diagnostics of electromechanical processes deals with the identification of changes in their states, what has been presented in the form of a sequence of intentional actions performed by means of the set of available machines and devices for a fixed period of time. After exceeding a certain value, the damage must be detected and identified. A diagnostic algorithm must detect and identify

* Electrical School No. 1 in Krakow, Kamieńskiego 49, 30-644 Kraków, tomczykmarcin@poczta.fm
** Cracow University of Technology, Warszawska 24, 31-155 Kraków, cnborowi@cyf-kr.edu.pl
*** The University of Bielsko-Biała, Willowa 2, 43-309 Bielsko-Biała, bo@borowik.info
in a relatively short time a fault that occurs early in the development phase (Korbicz, 2002). In mechanical connections containing backlashes, non-linear resilient-absorbing elements or damaged bearings – it is necessary to classify signals simultaneously in the time domain and depending on frequencies using transformation methods, to be able to study its spectral properties (Duda, 2007). The adaptive, time-frequency distribution of signals waveform processing has a number of important, scalable properties, relating both to time and to frequency, analyzing the relationship between the function being studied and its transformation coefficients (Doniec, 2010).

During the period of the last several decades more and more scientific works have been appearing in the literature presenting methods of industrial structures damage diagnostics using a time-frequency analysis and neural networks. It is worthy to mention some of them:

- presentation of diagnostics of various types of faults of induction motor by means of packet wavelet analysis (Kowalski, 2006),
- presentation of a new detection technique and the method of classifying faults of induction motors by means of current and dispersed stator stream analysis (Ishkova & Vitek, 2016),
- using a discrete wavelet transform of the current stator envelope for detection of spiral short circuits in an induction motor, in the initial phase of failure (Wolkiewicz & Kowalski, 2015),
- application of statistical features of wavelet distribution coefficients to a neural network training, using a backpropagation algorithm (Yayakumar, Thangavel & Elango, 2015),
- detection of a stator winding fault, using a combination of a wavelet transform and a neuro-fuzzy identifier (Farronato et al., 2005).

2. METHODOLOGY AND RESEARCH OF THE DIAGNOSTIC ALGORITHM FOR FAULT IDENTIFICATION

Diagnostic tests have been carried out for the nominal conditions of an induction motor whose model has been built in a stationary coordinate system related to the stator (model $\alpha$, $\beta$, 0). It is assumed that the load of the induction motor is a working machine of the form of a dynamic mass-absorbing-resilient element. Figure 1 shows in a simplified form a diagram of the connection of a working machine with the induction motor. The tested width of the backlash zone occurs between the rod of the induction motor drive and a working machine drive wheel. The backlash results from the line slip of the dynamic-absorbing-resilient dynamic element on the surface of the working machine drive wheel.
Fig. 1. Diagram of the dynamic mass-absorbing-resilient element, which has been connected to the used in the tests induction motor. The diagram shows the inertia moment of the masses reduced on the motor shaft.

The tests have been carried out within the MATLAB / Simulink environment, using the following parameters of the induction motor (parameters of its substitute circuit are expressed in relative units): stator relative resistance $r_s = 0.059 \, [\Omega]$, rotor relative resistance $r_w = 0.048 \, [\Omega]$, relative reactance of the dispersion circuit stator $x_s = 1.92 \, [\Omega]$, relative reactance of the dispersion circuit rotor $x_w = 1.92 \, [\Omega]$, relative reactance of the dispersed circuit $x_m = 1.82 \, [\Omega]$, $w = x_s \times x_w - x_m \times x_m = 0.374$, mechanical time constant $T_m = 0.86 \, [s]$.

3. IDENTIFICATION TESTS OF THE INERTIA MOMENT VALUES IN THE ELECTROMECHANICAL SYSTEM CONTAINING VISCOS FRICTION

The tests have been carried out in four test groups, with the following four different values of apparent viscosity coefficient $\eta_k$: 0.0125 [Pa s ^ 1], 0.025 [Pa s ^ 1], 0.0375 [Pa s ^ 1] and 0.05 [Pa s ^ 1]. Each test group contained six cases with different backlash zone width values. Results of simulations for all physical quantities and for every case of change of inertia moment of reduced masses and connected stiffly with the induction motor drive rotor $J_s$ have been written in the matrix $M_{[7,2048]}$. Elements of the matrix $M$ have been written for each consistency coefficient value $\eta_k$ (i.e. for apparent viscosity). Value of an inertia moment $J_s$ have been determined as the percentage in relation to its nominal value $J_s$, up with value $A\%$ and down with value $C\%$. Formal changes of inertia moment $J_s$ have been written in matrix "moments" in the following order: $moments = [\text{nominal value of inertia moment} (J_s = 0.862), A=2.5\% (J_s = 0.884), A=5\% (J_s = 0.905), A=8\% (J_s = 0.932), A=21\% (J_s = 1.045), C=2.5\% (J_s = 0.841), C=5\% (J_s = 0.819)]$. 

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In each of the six cases of changes of backlash zone width have been carried out simulation tests for seven inertia moment $J_s$ values. The backlash zone width values have been taken in sequence from the epsilon matrix, in the following order: $\epsilon = [0.0025, 0.00375, 0.005, 0.0075, 0.009, 0.01]$. All tests have been carried out for the creep index $n_i$ value equal 0.93.

The wavelet type and its order has been selected in such a way that the shape of the basic wavelet approximately would be adequate to the character of the transient course of the tested physical quantity, obtained as a result of a simulation for the case of the smallest backlash value. Based on the carried out tests the following selections of wavelets have been made for individual physical variables, with decomposition level 10:

a) linear acceleration of the induction motor drive $a_c$ – sym5,
b) electromagnetic moment of the induction motor drive $m_{el}$ – db6,
c) angular speed of the induction motor drive rotor $\omega_s$ – sym5,
d) linear acceleration of a mass $a_c$ – db6,
e) linear speed of a mass $v_c$ – sym5.

In these simulation tests, it has been assumed that the process of the electromechanical system dynamics testing in the backlash zone starts when the expression specified in the left part of the following inequality (1) is smaller than the right part of the below inequality:

\[ |\alpha_1 - \alpha_2| < \frac{\epsilon_{\alpha(i)}}{r} ; \quad i = 1,2,6, \tag{1} \]

where:
- $r$ – radius of the drive wheel of a working machine [m],
- $\epsilon_{\alpha(i)}$ – value that has been taken sequentially from the "epsilon" matrix and corresponding to the given backlash value in mechanical connection,
- $i$ – index number within the "epsilon" matrix.

The location change angle $\alpha_1$ for rod masses of the induction motor drive [rad/s] has been calculated using the formula: $\alpha_1 = \frac{x_1}{r}$, and the location change angle $\alpha_2$ of the dynamic mass-absorbing-resilient element mass [rad/s], has been calculated using the formula: $\alpha_2 = \frac{x_c}{r}$, where $x_c$ and $x_l$ – are linear distances, the first done by mass of the dynamic mass-absorbing-resilient element [m], and the second - done by rod mass of the induction motor drive [m].

After satisfying the condition determined by inequality described by formula (1) the load moment of dynamic mass-absorbing-resilient element is set to zero in the tested electromechanical system. For all tested physical quantities the matrix $M$
has contained 2048 samples chosen starting from the moment of obtaining backlash zone. Figure 2 presents the example of a course of the tested signal of the induction motor driver linear acceleration $a_i$ in a backlash zone.

Samples chosen according to the scheme presented on figure 2 have been written in sequence to the matrix $M$ for each executed simulation for a given backlash zone width.

![Fig. 2. Testing dynamics of the induction motor drive linear acceleration $a_i$ in a backlash zone, carried out during selected time range choices (samples of the tested signal)](image)

3.1. Processing of the three-layer neural network learned by the Levenberg-Marquardt algorithm with the use of a backpropagation method

Algorithm of a one-way neural network taught by means of the Levenberg-Marquardt algorithm using a backpropagation method and the cascade connections structure in this network – enables obtaining fast convergence at the relatively low computational complexity. In subsequent epochs of the network learning, if the calculated error value is smaller than the error in the previous (starting) point, then the coordinates of the new point will determine a new starting point with a corresponding set of weights that would have the ability to approximate the optimal parameters of the learned network.

Applied in the test neural network has contained three layers. To obtain the most profitable results of identification of backlash zone width in chosen physical quantities signals tests – an important element was to determine the number of neurons in the input layer, which represents the first layer of the applied neural network.

To obtain the appropriate number of samples, which determine input values as well as the proper output values of the neural network, there have been made many series of simulations and observations of results using different number of samples, which have been written in the matrix $M$. Finally, after the simulation tests execution, the last 400 samples have been chosen from the matrix $M$ and have been written in a separate matrix $M_1$. In every case considered in the entire this section the value of the consistency coefficient $\eta_k$ used in tests equals to 0.025. The same variables, which are used in many formulas, when their meaning stays the same, then it is explained only once during their first occurrence.
In case of all tested physical quantities every input value \( X \) in the first layer of the neural network has been transformed to the \([h_2, k_2]\) range of normalized values of the matrix \( M_j \) according to the formula:

\[
X_{(a)} = \left( \frac{M_j(t(X,t,a) - h_1)}{k_1 - h_1} \right) \cdot (k_2 - h_2) + h_2; \quad a = 1, 2, ..., 400; \quad e \in <1, 6>
\]  

(2)

where: \( M_j \) – values of the matrix \( M_j \), registered for testing in which consistency’s coefficient \( \eta_k = 0.025 \) and for assumed correct value of inertia moment \( J_s \),

\( h_1 \) and \( k_1 \) – the minimal and maximal value (respectively) of the matrix \( M_j \), for assumed correct value of inertia moment \( J_s \),

\( h_2 \) and \( k_2 \) – the initial and the final value of the range, which contains normalized values of the matrix \( M_j \),

\( e \) – an index of a column’s number in the "epsilon" matrix.

The set values \( T \) of the neural network have been calculated in a similar way as input values \( X \) of the network. A transformation to the \([h_2, k_2]\) range has been carried out using the following formula:

\[
T_{(a)} = \left( \frac{k_1 - M_j(t(X,t,a))}{k_1 - h_1} \right) \cdot (k_2 - h_2) + h_2; \quad \text{for } a = 1, 2, ..., 400; \quad e \in <1, 6>; \quad i = 1, 2, ..., 7
\]  

(3)

The beginning as well as the end of the range, to which values of the matrix \( M_j \) have been transformed – they represent the minimal (\( h_2 = \min(h_3, k_3) \) ) and the maximal (\( k_2 = \max(h_3, k_3) \) ) values, which have been determined for the two compared values of \( h_3 \) and \( k_3 \), where \( h_3 \) is the variable that has been determined using statistical parameters calculated for the matrix \( M_2 \) rows in testing, and \( k_3 \) is the variable determined using the difference between normalized median value of the tested matrix \( M \) row and the sum of values of variables \( h_3 \) and \( h_4 \).

Therefore the value of the variable \( h_3 \) has been calculated in the following way: \( h_3 = \min \left( m, \frac{h_5 - m_5}{k_5} \right) \), where: \( m_1 \) is the average value of the matrix \( M_2 \), \( h_5 \) is the minimal value of the matrix \( M_2 \), and \( k_5 \) the maximal value of the matrix \( M_2 \).

Statistical parameters \( m_1, h_5 \) and \( k_5 \) have been calculated for matrix \( M_2 \) rows for testing, for assumed backlash zone width. For all tested physical quantities values of matrix \( M_2 \) contains medians of rows in matrix \( M_3 \). Matrix \( M_j \) contains sorted in the ascending order values of rows of matrix \( M \) for testing, in which assumed consistency coefficient \( \eta_k \) has been set to 0.025.
To determine the value of the parameter $k_3$, it is necessary to perform its normalization, i.e. median of matrix’s tested row $M$ must be decreased by sum of parameter values $h_3$ and $h_4$. The normalization formula is as follows:

$$k_3 = \begin{cases} 
\frac{(m_2 - h_3)}{(k_5 - h_5)} - (h_3 + h_4) & \text{for } \max(m_5, m_6) = m_5 \\
\frac{(k_5 - m_2)}{(k_5 - h_5)} - (h_3 + h_4) & \text{for } \max(m_5, m_6) = m_6 
\end{cases}$$

(4)

where: $m_2$ – the value of median of matrix’s tested row $M$, $m_5$ – the average value of matrix $M_5$ for test, $m_6$ – the average value of matrix $M_6$ for test.

Value of the parameter $h_4$ is determined as the following minimum:

$$h_4 = \min \left( m_2, \frac{h_5}{k_5} - m_2 \right) .$$

For all tested physical quantities value of median $m_2$ is calculated for values of matrix $M_4$. This matrix contains sorted in the ascending order values of matrix $M$. Values of arithmetic means $m_5$ and $m_6$ were calculated for rows in matrices respectively $M_5$ and $M_6$ for testing. In case of all tested physical quantities values of matrices $M_5$ and $M_6$ are located in the range $[0, 1]$ and represent the result of the normalization process of $M_2$ matrix, carried out according to the below formulas:

$$M_{5(e)(i)} = \frac{(M_{2(e)(i)} - h_5)}{(k_5 - h_5)} ; \; e \in \{1,6\}; \; i = 1,2,...7$$

(5)

$$M_{6(e)(i)} = \frac{(k_5 - M_{2(e)(i)})}{(k_5 - h_5)} ; \; e \in \{1,6\}; \; i = 1,2,...7$$

(6)

where: $M_2$ – matrix containing sorted in the ascending order values of matrix $M$ rows for testing, in which consistency’s coefficient $\eta = 0.025$, $h_5$ and $k_5$ – are respectively the minimal and maximal value of the matrix $M_2$.

The initial values of weights $W_{11}, W_{12}, W_{21}, W_{31}$ and $W_{32}$ – have been calculated in the following way: $W_{1(i)} = M_{7(i)}$, $W_{12(i)} = M_{7(i)}$, $W_{2(i,j)} = M_{7(i)}$, $W_{3(i)} = M_{7(i)}$, $W_{32(i)} = M_{7(i)}$, for indexes $i,j = 1,2,...,7$, where $M_7$ – matrix contains maximum values of rows respectively matrix $M_8$ as well $M_9$. 

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Therefore values of the matrix $M_7$ are calculated for test executed with consistency’s coefficient $\eta_k = 0.025$, according to the following formula:

$$M_{7(i)} = \max \left( \max \left( M_{8(i,j)} \right), \max \left( M_{9(i,j)} \right) \right)$$

$i = 1,2,\ldots,7$; $j = 1,2,\ldots,2048$  \hspace{1cm} (7)

The initial value of $W_{13}$ weight represents the arithmetic mean $m_7$, calculated using $M_7$ matrix:

$W_{13} = m_7$.

Values of matrices $M_8$ and $M_9$ are located in the range $[0, 1]$ and represent values of matrix $M$, have been obtained as a result of the normalization process of all considered physical quantities used in testing, according to the below formulas:

$$M_{8(i,j)} = \frac{(M_{e(i,j)} - h_e)}{(k_e - h_e)}; \quad e \in <1,6>; i = 1,2,\ldots,7; j = 1,2,\ldots,2048;$$

$$M_{9(i,j)} = \frac{(k_e - M_{e(i,j)})}{(k_e - h_e)}; \quad e \in <1,6>; i = 1,2,\ldots,7; j = 1,2,\ldots,2048;$$

where: $h_e$ and $k_e$ – are respectively the minimal and the maximal value of the matrix $M$ used for testing.

In executed simulations some parameters have been calculated especially only for testing, in which assumed consistency coefficient $\eta_k$ has been set up to the value 0.025. It was applied for the following parameters: $h_1, h_3, h_5, h_6, k_1, k_5, k_6, m_1, m_5$ and $m_6$.

The initial values of matrix of biases for neural network first and hidden layer have been set to 1, i.e. $B_{1(i)} = 1$; $f = 1,2; i = 1,2,\ldots,7$, where $f$ is the number of neural network layer applied in test. It has been assumed that the initial value of bias $B_{1}$ in the output layer was also set to 1. Output signal $Y_i$ from the first layer of the neural network have been calculated as follows:

$$Y_{1(a,i)} = X_{1(a)} * W_{1(i)} + B_{1(i)}; \quad a = 1,2,\ldots,400; i = 1,2,\ldots,7;$$

where $B_{1}$ – is the matrix of biases in the neural network input layer.

During executed tests a linearity of neurons activation function has been assumed. The output signals $Y_i$ from the hidden (i.e. the second) layer of the neural network, have been calculated as follows:
\[ Y_{2(a,j)} = \sum_{j=1}^{7} Y_{1(a,j)} \ast W_{2(b,j)} + X_{1(a)} \ast W_{12(i)} + B_{2(i)}; \quad a = 1,...,400; \quad i = 1,...,7, \quad (11) \]

where \( B \) is the matrix of biases in the hidden layer of the neural network. The output signals \( Y_{3} \) of the output layer in the neural network have been obtained in the similar way, as in the previous layers, i.e.:

\[ Y_{3(a)} = \sum_{i=1}^{7} Y_{2(a,j)} \ast W_{32(i)} + \sum_{i=1}^{7} Y_{1(a,j)} \ast W_{13(i)} + W_{33(i)} + B_{3}; \quad (12) \]

for \( a = 1,...,400, \)

where \( B_{3} \) is the value of bias in output layer of neural network.

Adaptation of weights and biases have been carried out for each epoch of neural network learning by means of updating values in matrix \( W_{4} \), containing all weights and biases that have been obtained using the following formula:

\[
W_{4(a)} = \begin{bmatrix}
W_{4(1)} & W_{4(2)} & \cdots & W_{4(7)} \\
W_{4(1)} & W_{4(2)} & \cdots & W_{4(7)} \\
\vdots & \vdots & \ddots & \vdots \\
W_{4(1)} & W_{4(2)} & \cdots & W_{4(7)} \\
W_{4(1)} & W_{4(2)} & \cdots & W_{4(7)} \\
\end{bmatrix} = \begin{bmatrix}
\frac{W_{4(1)}}{a = 1, 2 \ldots 7; \quad i = 1, 2 \ldots 7} & \frac{W_{4(2)}}{a = 8, 9 \ldots 14} \\
\frac{W_{4(1)}}{a = 1, 2 \ldots 7} & \frac{W_{4(2)}}{a = 1, 2 \ldots 7} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{W_{4(1)}}{a = 1, 2 \ldots 7} & \frac{W_{4(2)}}{a = 1, 2 \ldots 7} \\
\frac{W_{4(1)}}{a = 1, 2 \ldots 7} & \frac{W_{4(2)}}{a = 1, 2 \ldots 7} \\
\end{bmatrix} \quad (13)
\]

Values of matrix \( W_{4} \) have been changed in the following way (Rusiecki, 2007):

\[ W_{4(i)} = W_{4(i)} - [H_{(i,j)}]^{\dagger} \cdot \nabla G_{(i,j)}; \quad a = 1,...,400; \quad i = 1,...,93; \quad j = 1,...,93, \quad (14) \]

where \( \nabla G \) – is a matrix of neural network gradient, and \( H \) – is the approximated Hessian matrix, applied to the Levenberg-Marquardt algorithm.
The gradient matrix $\nabla G$ is determined according to the formula (Rusiecki, 2007):

$$\nabla G_{(a)} = J_{(a)}^T \cdot E_{(a)}^T; \quad a = 1,2,\ldots,400; \quad i = 1,2,\ldots,93. \quad (15)$$

The Hessian matrix $H$ is calculated in the following way (Rusiecki, 2007):

$$H_{(i,j)} = J_{(a,i)}^T \cdot J_{(a,j)} + l \cdot I_{(j,k)}; \quad a = 1,2\ldots,400; \quad i = 1,2\ldots,93; \quad j = 1,2\ldots,93 \quad (16)$$

where: $I$ – identity matrix assumed as diagonal,

$J$ – Jacobian matrix of neural network,

$l$ – coefficient of neural network learning, and belongs to the $[0,1]$ range.

Values of the neural network Jacobian matrix $J$ have been calculated according to following formula (Rusiecki, 2007):

$$J_{(a,i)} = \begin{bmatrix}
\frac{\partial E_{(1)}}{\partial W_{4(1)}} & \frac{\partial E_{(1)}}{\partial W_{4(2)}} & \ldots & \frac{\partial E_{(1)}}{\partial W_{4(93)}} \\
\frac{\partial E_{(2)}}{\partial W_{4(1)}} & \frac{\partial E_{(2)}}{\partial W_{4(2)}} & \ldots & \frac{\partial E_{(2)}}{\partial W_{4(93)}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial E_{(400)}}{\partial W_{4(1)}} & \frac{\partial E_{(400)}}{\partial W_{4(2)}} & \ldots & \frac{\partial E_{(400)}}{\partial W_{4(93)}}
\end{bmatrix}; \quad (17)$$

where: $E$ – matrix containing values of the neural network errors,

$W_d$ – matrix of the neural network weights and biases.

In order to provide the appropriate approximation of the Hessian matrix $H$, the following diagonal identity matrix $I$ has been was provided:

$$I_{(i,j)} = \begin{cases} 
1 & \text{for } i = j \quad \text{for } i = 1,2,\ldots,93; \quad j = 1,2,\ldots,93; \\
0 & \text{for } i \neq j
\end{cases} \quad (18)$$

The coefficient of neural network learning $l$ changes in every given epoch of this network learning process depending on the value of the mean square error ($MSE$) and this is shown by the below formula:
\[ l_{(x+1)} = \begin{cases} l_{(x)} \ast l_1; & \text{for } MSE_{(x)} > MSE_{(x-1)} \\ l_{(x)} \ast l_2; & \text{for } MSE_{(x)} < MSE_{(x-1)} \end{cases}, \quad (19) \]

where:  
- \( g \) – the number of a given epoch of this neural network learning process,
- \( l_1 \) – the first beginning coefficient, set to 10,
- \( l_2 \) – the second coefficient set to 0.1.

In the output layer of the neural network values of errors on neurons have been stored in matrix \( E \). They have been calculated on the basis of the difference between the output values \( Y \) and set values \( T \), i.e.: \( E(a) = (Y_{3(a)} - T_{a}) \), where \( a = 1, 2, ..., 400 \). The mean square error (MSE) has been calculated according to the formula: \( \frac{1}{400} \sum_{a=1}^{400} (E(a))^2 \) and compared with \( \delta \), i.e. with the earlier experimentally assumed value, used for stopping the learning process of the neural network. In other words, when the condition of \( MSE < \delta \) has been satisfied, the neural network learning process has been finished.

For the earlier assumed value of consistency coefficient \( \eta_k = 0.025 \) and backlash zone width there have been created pattern matrices \( W \) (for every group of tests one matrix). Each such pattern matrix with set up earlier value of \( \eta_k = 0.025 \) and backlash zone width – was accepting changing respectively values of the inertia moment \( J \). In this way has been obtained a matrix with a dimension 7 by 7, and this can be expressed by the formula:

\[ W_{w(i,j)} = \frac{W_{32(j)}}{h_3}; \quad \text{for } e \in <1,6> \text{ and } i, j = 1, 2, ..., 7 \quad (20) \]

where: \( h_3 \) – is the value of the variable necessary to determine the range, in which would be placed calculated values of matrix \( X \) and \( T \).

In the same way, with the earlier set up a backlash zone width and for \( e \in <1,6> \text{ and } j = 1, 2, ..., 7 \), it has been determined values of the matrix \( W_6 \) of dimensions 1 by 7: \( W_{b(e)(j)} = \frac{W_{32(j)}}{h_5} \).

Correct identification of the value of the inertia moment in the executed test for every physical quantity is possible for having assumed earlier values of these quantities can be obtained using the values of the matrix \( G \) calculated according to the formula:
\[ G_{(i)} = \sum_{j=1}^{n} |W_{i(j)} - W_{e(j)}|; \quad e \in \{1, 6\}; \quad i = 1, 2, \ldots, 7; \quad (21) \]

Index \( nr_6 \) (for \( nr_6 \in \{1, 7\} \)) in matrix \( G \) determines the column number in the matrix "moments", which contains the correct value of inertia moment. The value of index \( nr_6 \) (\( nr_6 \in \{1, 7\} \)) for \( i = 1, 2, \ldots, 7 \) in matrix \( G \) has been determined using the following minimum function: \( G_{(nr_6)} = \min(G_{(i)}) \).

Therefore, the number \( i \) of a column in the matrix "moments" refers to the corresponding to it index \( nr_6 (i = nr_6) \).

### 3.2. Simulation results of the algorithm of the identification of the moment of inertia value while changing the width of the backlash zone in the electromechanical system, using a three-layer neural network

In the below tables, using the bold font have been presented the resulting final results of calculations of the matrix \( G \), while in the column named Test parameters there have been placed the assumed set up earlier values, like the widths of the backlash zone, adopted in the process of identifying the moment of inertia of the reduced masses and connected stiffly with the rotor of the induction motor driver. The values of the \( G \) matrix presented in the Table 1 through 4 – are the correct results obtained finally in the process of identifying the fault number. Pattern matrices \( W \) have been created for analyzes, and in them it was assumed both, the specific value of the learning coefficient \( l \) of the neural network, as well as the value \( \delta \) – as the value of the accepted error, which has allowed to stop the neural network learning process. On the basis of the executed simulations and analyzes of its results it is noticeable that obtainment of correct results of the identification number of a fault for all tested physical quantities is possible, while the following condition is satisfied: execution of a simulation with the neural network learning coefficient \( l \) changing within the range from 0.1 to 0.9, with the value of \( \delta \) equal to \( 10^{-4} \), i.e. the parameter controlling the stopping time of the neural network learning process. Results that have been shown in Table 1 through 4 illustrate this fact independently from the obtained number of epochs of the neural network processing.
Tab. 1. Exemplified results of tests in matrix $G$ for angular speed of the rotor of the induction motor drive $\omega_s$

<table>
<thead>
<tr>
<th>Test parameters</th>
<th>Results</th>
<th>Test parameters</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>inertia moment</td>
<td>0.0014</td>
<td>inertia moment</td>
<td>0.0019</td>
</tr>
<tr>
<td>$J_s = 0.819$,</td>
<td>0.0032</td>
<td>$J_s = 0.819$,</td>
<td></td>
</tr>
<tr>
<td>backlash zone $= 0.0025$,</td>
<td>0.0053</td>
<td>backlash zone $= 0.0025$,</td>
<td></td>
</tr>
<tr>
<td>$\eta_k = 0.05$, epochs = 7,</td>
<td>0.0084</td>
<td>$\eta_k = 0.05$, epochs = 6,</td>
<td></td>
</tr>
<tr>
<td>$l = 0.9$, $\delta = 10^{-5}$</td>
<td>2.8045</td>
<td>$l = 0.9$, $\delta = 10^{-4}$</td>
<td>4.8939</td>
</tr>
<tr>
<td></td>
<td>0.0007</td>
<td></td>
<td>0.0007</td>
</tr>
<tr>
<td>inertia moment</td>
<td>0.0063</td>
<td>inertia moment</td>
<td>0.0011</td>
</tr>
<tr>
<td>$J_s = 0.932$,</td>
<td>0.0053</td>
<td>$J_s = 0.932$,</td>
<td></td>
</tr>
<tr>
<td>backlash zone $= 0.0075$,</td>
<td>0.0034</td>
<td>backlash zone $= 0.0075$,</td>
<td></td>
</tr>
<tr>
<td>$\eta_k = 0.0375$, epochs = 7,</td>
<td><strong>0.0004</strong></td>
<td>$\eta_k = 0.0375$, epochs = 6,</td>
<td></td>
</tr>
<tr>
<td>$l = 0.9$, $\delta = 10^{-5}$</td>
<td>1.6930</td>
<td>$l = 0.1$, $\delta = 10^{-4}$</td>
<td><strong>0.0005</strong></td>
</tr>
<tr>
<td></td>
<td>0.0070</td>
<td></td>
<td>0.0009</td>
</tr>
<tr>
<td></td>
<td>0.0076</td>
<td></td>
<td>0.0013</td>
</tr>
</tbody>
</table>

| inertia moment  | 0.0129  | inertia moment | 0.0037  |
| $J_s = 0.884$,  | **0.0052** | $J_s = 0.884$,  |
| backlash zone $= 0.01$, | 0.0216 | backlash zone $= 0.01$, |
| $\eta_k = 0.0125$, epochs = 7, | **1.6877** | $\eta_k = 0.0125$, epochs = 6, |
| $l = 0.9$, $\delta = 10^{-5}$ | **1.4747** | $l = 0.1$, $\delta = 10^{-4}$ | **0.0001** |
|                 | 0.0254  |                 | 0.0082  |
|                 | 0.0300  |                 | 0.0027  |

Tab. 2. Exemplified results of tests in matrix $G$ for linear acceleration of the mass $a_c$

<table>
<thead>
<tr>
<th>Test parameters</th>
<th>Results</th>
<th>Test parameters</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>inertia moment</td>
<td>1.4902</td>
<td>inertia moment</td>
<td>4.5547</td>
</tr>
<tr>
<td>$J_s = 1.045$,</td>
<td>1.4776</td>
<td>$J_s = 1.045$,</td>
<td></td>
</tr>
<tr>
<td>backlash zone $= 0.009$,</td>
<td>1.4566</td>
<td>backlash zone $= 0.009$,</td>
<td></td>
</tr>
<tr>
<td>$\eta_k = 0.0375$, epochs = 6,</td>
<td><strong>0.0864</strong></td>
<td>$\eta_k = 0.0375$, epochs = 5,</td>
<td></td>
</tr>
<tr>
<td>$l = 0.9$, $\delta = 10^{-5}$</td>
<td>0.5744</td>
<td>$l = 0.9$, $\delta = 10^{-4}$</td>
<td><strong>0.0482</strong></td>
</tr>
<tr>
<td></td>
<td>1.4958</td>
<td></td>
<td>4.5754</td>
</tr>
<tr>
<td></td>
<td>1.5023</td>
<td></td>
<td>4.5910</td>
</tr>
<tr>
<td>inertia moment</td>
<td>0.0129</td>
<td>inertia moment</td>
<td>0.0037</td>
</tr>
<tr>
<td>$J_s = 0.884$,</td>
<td><strong>0.0052</strong></td>
<td>$J_s = 0.884$,</td>
<td></td>
</tr>
<tr>
<td>backlash zone $= 0.01$,</td>
<td>0.0216</td>
<td>backlash zone $= 0.01$,</td>
<td></td>
</tr>
<tr>
<td>$\eta_k = 0.0125$, epochs = 7,</td>
<td><strong>1.6877</strong></td>
<td>$\eta_k = 0.0125$, epochs = 6,</td>
<td></td>
</tr>
<tr>
<td>$l = 0.9$, $\delta = 10^{-5}$</td>
<td><strong>1.4747</strong></td>
<td>$l = 0.1$, $\delta = 10^{-4}$</td>
<td><strong>0.0001</strong></td>
</tr>
<tr>
<td></td>
<td>0.0254</td>
<td></td>
<td>0.0082</td>
</tr>
<tr>
<td></td>
<td>0.0300</td>
<td></td>
<td>0.0027</td>
</tr>
</tbody>
</table>

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Tab. 3. Exemplified results of tests in matrix $G$ for linear acceleration of the induction motor drive $a_t$

<table>
<thead>
<tr>
<th>Test parameters</th>
<th>Results</th>
<th>Test parameters</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>inertia moment</td>
<td>0.0526</td>
<td>inertia moment</td>
<td>0.0506</td>
</tr>
<tr>
<td>$J_s = 0.884$,</td>
<td>1.0286</td>
<td>$J_s = 0.884$,</td>
<td>2.2530</td>
</tr>
<tr>
<td>backlash zone =</td>
<td>0.1911</td>
<td>backlash zone =</td>
<td>1.8502</td>
</tr>
<tr>
<td>0.0075, $\eta_k$=</td>
<td>3.2197</td>
<td>0.0075, $\eta_k$=</td>
<td>8.0211</td>
</tr>
<tr>
<td>$0.0125$, epochs =</td>
<td>0.0094</td>
<td>epochs = 6, l = 0.9, $\delta = 10^{-5}$</td>
<td>0.0075</td>
</tr>
<tr>
<td>$0.0170$</td>
<td>0.0023</td>
<td>$0.0094$</td>
<td>0.0165</td>
</tr>
</tbody>
</table>

| inertia moment  | 3.5903  | inertia moment  | 3.0191  |
| $J_s = 0.905$,  | 3.1856  | $J_s = 0.905$,  | 5.2248  |
| backlash zone = | 10.8113 | backlash zone = | 13.9348 |
| 0.009, $\eta_k$= | 2.6810  | 0.009, $\eta_k$= | 3.0097  |
| $0.05$, epochs = | 2.7129  | epochs = 4, l = 0.1, $\delta = 10^{-4}$ | 3.0134  |
| $2.6956$         |         | $0.0023$        | 3.0110  |

Tab. 4. Exemplified results of tests in matrix $G$ for Electromagnetic moment of the induction motor drive $m_{el}$

<table>
<thead>
<tr>
<th>Test parameters</th>
<th>Results</th>
<th>Test parameters</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>inertia moment</td>
<td>0.0182</td>
<td>inertia moment</td>
<td>0.0198</td>
</tr>
<tr>
<td>$J_s = 0.841$,</td>
<td>0.0454</td>
<td>$J_s = 0.841$,</td>
<td>0.0638</td>
</tr>
<tr>
<td>backlash zone =</td>
<td>1.5448</td>
<td>backlash zone =</td>
<td>0.1773</td>
</tr>
<tr>
<td>0.005, $\eta_k$=</td>
<td>1.0588</td>
<td>0.005, $\eta_k$=</td>
<td>3.0455</td>
</tr>
<tr>
<td>$0.05$, epochs =</td>
<td>0.0864</td>
<td>epochs = 6, l = 0.9, $\delta = 10^{-5}$</td>
<td>0.1373</td>
</tr>
<tr>
<td>$0.0011$</td>
<td>0.0081</td>
<td>$0.0011$</td>
<td>$0.0020$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0101</td>
</tr>
</tbody>
</table>

| inertia moment  | 1.7720  | inertia moment  | 5.2338  |
| $J_s = 1.045$,  | 1.7384  | $J_s = 1.045$,  | 5.2311  |
| backlash zone = | 1.6812  | backlash zone = | 5.2255  |
| 0.0025, $\eta_k$= | 0.3638  | 0.0025, $\eta_k$= | 0.0258  |
| $0.0375$, epochs = | 0.0269  | epochs = 5, l = 0.1, $\delta = 10^{-4}$ | $0.0063$ |
| $0.0375$, $\eta_k$= | 1.7914  | $0.0375$, epochs = | 5.2351  |
| $0.0375$, $\delta = 10^{-5}$ | 1.8023  | $0.0375$, epochs = | 5.2363  |
4. CONCLUSIONS

The presented paper describes a fault detection system containing a neural network trained using the Levenberg-Marquardt algorithm together with the adapting back error propagation method. The system has been applied to the identification of the inertia moment of reduced masses and connected stiffly with the induction motor drive rotor $J_s$.

The process of the identification was executed for certain time periods. These ranges have been obtained as the result of changes of the width of the backlash zone by means of time-frequency methods with the multistage decomposition of the signal for the complex electromechanical system, connected with a dynamic mass-absorbing-resilient element.

The application of the wavelet-neuron method strongly affects the efficiency of the analysis of non-stationary signals in the executed researches, effectively limiting the dangerous consequences of appearing fault in the initial phase of its development.

On the basis of the executed tests, we can notice that ensuring appropriate changes in parameters in a system containing non-zero backlash zones causes results to be much more effective in the detection and identification of the fault. These parameters are: coefficient of neural network learning and the fixed stopping value of the neural network learning process.

REFERENCES


