electro-mechanically coupling, activator,
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AN INFLUENCE OF THE d_{311} EFFECT ON THE BEHAVIOR OF THE CANTILEVER BEAM-SHAPED PIEZOELECTRIC ACTIVATOR MADE OF TWO LAYERS OF PVDF WITH INVERSE POLARITY

Abstract

In the present work an example of the numerical modeling of electro-mechanical coupling of the activator in the form of cantilever beam made of two layers PVDF subjected to an inverse polarity is described. Calculations were done for static deflections at the selected voltage levels and for modal analysis with an included and excluded piezoelectric effect. The effect of active material on the behavior of the beam was examined. In order to validate the model, results of the static calculation were compared with the strict analytical solution and the results presented by other authors.

1. INTRODUCTION

Activators are very important parts of many micro-electromechanical systems (MEMS). Starting from 1980 many solutions were presented of the activators, which used different effects, for example electrostatic, electromagnetic, piezoelectric, shape memory alloy (SMA). Activators with the piezoelectric effect are most popular, they have wide application - among others, in electrical fans, microphones, printers, engines, pumps, control systems and injectors [1,2]. A characteristic feature of piezoelectric actuators is their high potency and very

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small displacements they can trigger in a system. Force and displacement are dependent on the applied voltage. In many research a piezoelectric polymer polyvinylidene fluoride (PVDF) is used because it is easy to be formed and gives a possibility of constructing very small activators. Additional advantage of the PVDF polymer is that the material is particularly examined and data are available to describe the mechanical and electric properties. In addition, in the literature you can find results of experimental, analytical or numerical research for elements made from this material [1,2]. The literature addressed to cantilever beams made from the polymer PVDF type beams like bimorph are very extensive [[3]-[9]].

The aim of this paper is to describe influence of an electromechanical coupling of static and dynamic behavior of the activator in the form of cantilever beam made of two layers of polymeric piezoelectric polyvinylidene fluoride (PVDF) with inverse polarity. In order to achieve the peak, effect of electro-mechanical combination of both layers must be rigid. For this purpose, numerical simulations were performed with the Finite Element Method (FEM). The results of static calculations were validated with those published by other authors [8-9] and received from the linear analytical model. The choice of 'benchmarks' yielded from the completeness of data available, what was necessary for numerical simulations. In the second stage the validated model was used to determine natural frequencies and modes for bimorph-like beams with piezoelectric effect (open system) and without the piezoelectric effect (closed system).

In addition, natural frequencies and mode shapes determined for the open system were compared with the results for the beam made of the material having mechanical properties the same as PVDF, but without the piezoelectric effect.

2. ELECTRO-MECHANICALL COUPLING EFFECT/ PIEZOELECTRIC TRANSDUCERS

The PZT activators use the piezoelectric effect, which consists in the phenomenon that the piezoelectric element located in electric field undergoes deformation, while under mechanical load acting on its surface the active material generates electrical charge.

According to the above described properties of the PZT, one can write the following constitutive relation [1]:

$$\{\varepsilon\} = [S]\{\sigma\}+[d]\{E\} \tag{1}$$

where: $\{\varepsilon\}$ – the strain vector,
$\{\sigma\}$ – the stress vector,

{E} – components of the electric field strength,
[S] – the compliance matrix,
[d] – the matrix of piezoelectric coefficients.

Further it was assumed that the intensity of the electric field E_3 acts only along the x_3 axis (Fig. 1). Its value can be determined for a specified voltage U applied on upper and lower surfaces of beam from dependencies:

$$E_3 = U/h \tag{2}$$

where: U – voltage applied to the system,
h – thickness of the element.

Figure 1 shows piezoelectric effect d_{311}, when examined strain or stress in axis direction x_1 induced by E_3.

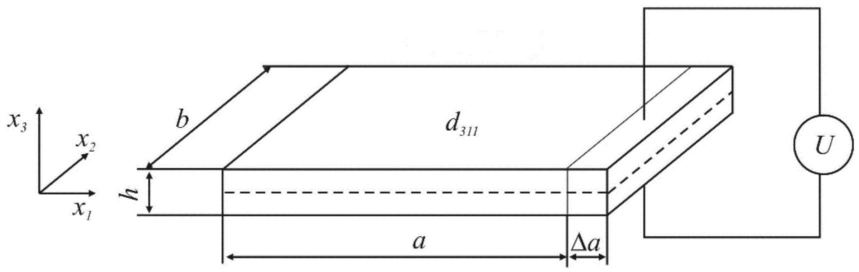


Fig. 1. The d_{311} effect for piezoelectric material [source: own study]

In this case the extension Δa in axis directions x_1 is:

$$\Delta a = d_{311}E_3a = d_{311}Ua/h \tag{3}$$

where: d_{311} – the electromechanical coupling coefficients in directions 1,
a – length of the element.

The ε_{11} strain component and the σ_{11} stress component in the direction x_1 are:

$$\varepsilon_{11} = d_{311}U/h$$
$$\sigma_{11} = d_{311}U\,E_{11}/h = e_{311}\,U/h \tag{4}$$

respectively; here:
E_{11} – Young's modulus,
e_{311} – the piezoelectric coefficients in the directions 1.

3. RESEARCH OBJECT

Object of analysis was an activator of the "bimorph" type having the form of cantilever beam. It was further assumed that the material of the beam was isotropic. A schematic diagram and dimensions of the examined system are shown in Fig. 2.

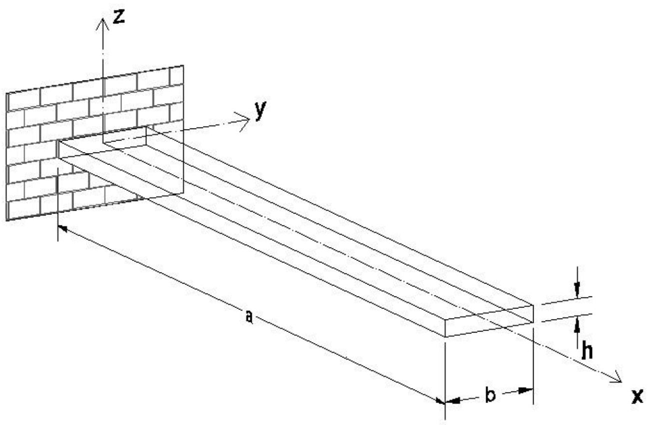


Fig. 2. The activator in the form isotropic cantilever beam [source: own study]

The activator was composed of two combined beams of piezoelectric material PVDF inversely polarized.

The accepted geometrical dimensions of the beam were as follows (Figure 2): *a* – the length of the beam was 100 mm, *b* – beam width – 5 mm *h* – beam thickness – 1 mm (each piezoelectric layer had a thickness of 0.5 mm). The density of the PVDF polymer was $\rho = 1600$ kg/m^3. Other material data of the active elements are given in Table 1. Beam dimensions were the same as in the works [[7]-[8]].

Table 1. The PVDF's mechanical and piezoelectric properties [[8]-[10]]

Mechanical properties				
Young's modulus $E_{11}=E_{22}=E_{33}$		Shear modulus $G_{12}=G_{13}=G_{23}$		Poisson's ratio $\nu_{12}=\nu_{13}=\nu_{23}$
2GPa		1GPa		0
Piezoelectronic propetries				
Piezoelectric constant				Electric permittivity
$e_{311}=e_{322}$	e_{333}	$d_{311}=d_{322}$	d_{333}	$\chi_{11}=\chi_{22}=\chi_{33}$
0.046 C/m2	0	0.023 nm/V	0	0.1062 nF/m

4. NUMERICAL MODEL OF THE ACTIVATOR

The numerical (FEA) model of the cantilever beam was constructed by using the C3D20RE type solid elements. They were 20-node 2nd order (with a square shape function) elements, having three translational degrees of freedom at each node and one extra degree of freedom associated with the piezoelectric properties.

All elements used a reduced integration method (elements with full integration caused over rigidity of the structure in test simulations). The simulations were performed using a commercial package Abaqus [[11]].

The mechanical boundary conditions of the numerical model were realized by restraining the nodes located on one end of the beam all the translational degrees of freedom and modeling this way the beam's restraint. Density of the mesh was right.

The combination of the PVDF layers was realized by defining interactions as "TIE", what resulted in linking the degrees of freedom of nodes in contact on the appropriate surfaces of the model. The developed FEM numerical model of the "bimorph" beam is shown in Figure 3.

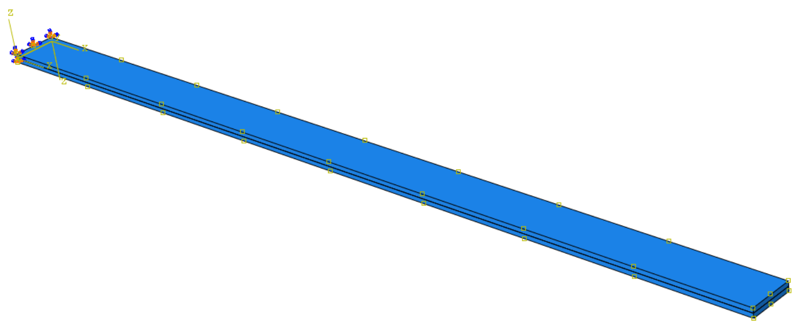


Fig. 3. The FEM model of the "bimorph" beam [source: own study]

In the numerical model, the electric field as the beam's load was introduced through boundary conditions by defining the value of the electrical potential both on the upper and the lower surface of the beam. In subsequent simulations the potential equal zero was applied on the bottom surface. On the upper surface of the beam a target value of the potential was applied (expressed in volts). For the examined system the load-like electric field applied in the direction 3 resulted in a dimension change in the direction 1; this was the examination of the d_{311} effect. Note, that the two parts of the activator were inversely polarized, so one part extending, while the second one shrank, and the resulting moment caused the beam's bending. This allowed the deflection measurement of the beam in static regime. In dynamic research two cases were considered: an open and a closed system. The open system was the case in which the bottom surface was treated with 0V and on the upper surface no voltage was applied. The closed system had a short circuit across the two surfaces.

5. STATIC TESTS

In order to validate the FEA model calculations of the beam's center line were performed at 1V voltage on the upper surface, while on the lower surface the voltage equaled 0V. The solution to this problem is a very popular benchmark known from the literature [[3]-[9]]. Figure 4 shows the deflection of the beam under the applied electrical load.

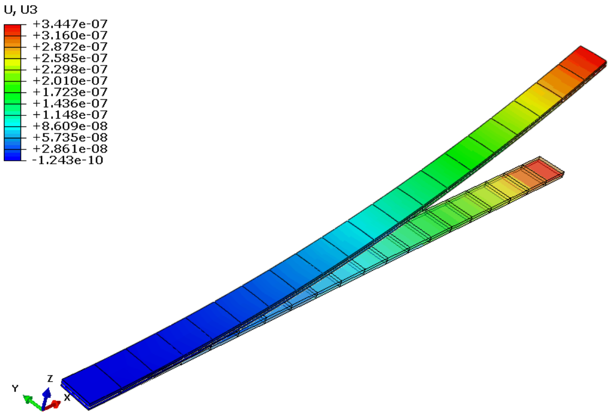


Fig. 4. Deflection of the beam under 1V voltage [source: own study]

In addition, by solving a linear equation of deflection line with analytical method, the center line deflection was determined. The resulting solution is correct for small deflections. The equation of the beam's center line deflection can be written as [[12]]:

$$E_{11} J_{x_2} \frac{d^2 u_3}{dx_1^2} = M_{x_2} \tag{5}$$

where: J_{x2} – second moment of area for the beam's cross-section,
u_3 – deflection in the direction 3,
M_{x2} – bending moment.

For the analyzed present case the bending moment created when the voltage U was applied between the upper and the bottom surface of the beam, which caused the expansion of one part and shortening of the second part, in accordance with the equation (4) is equal:

$$M_{x_2} = \sigma_{11} b \left(\frac{h}{2} \right)^2 = \frac{d_{311} U E_{11}}{4} bh = \frac{e_{311} U}{4} bh \tag{6}$$

Substituting the bending moment (6) into equation (5) gives:

$$u_3 = \frac{3}{2} d_{311} U \left(\frac{x_1}{h} \right)^2 = \frac{3}{2} \frac{e_{311}}{E_{11}} U \left(\frac{x_1}{h} \right)^2 \tag{7}$$

where: x_1 – distance from the fixed end.

All the results are presented in Table 2.

Tab. 2. The static deflection of the PVDF beam u3 [µm] for voltage equal 1V

x_1 [mm]	Theory according to (7)	Abaqus C3D20RE	Pablo et al [[8]]	Jiang et al [[9]]
20	0.0138	0.0140505	0.0137	0.0136
40	0.0552	0.0553451	0.0549	0.0545
60	0.1242	0.1241730	0.1235	0.1226
80	0.2208	0.2205330	0.2197	0.2180
100	0.3450	0.3444270	0.3433	0.3410

The results of the FEM numerical simulations differ by about 0.2% as compared with the results obtained with analytical calculations (according to the formula (7)) and about 0.5% with respect to the results presented in the papers [[8]-[9]]. This shows a very good compatibility of the obtained results with those available in the literature.

In the second stage, the FEM simulations for voltages from 50 to 200V were performed. The deflections obtained numerically were compared with the results of analytical calculations and shown in Figure 5. The results in the whole voltage range are similar.

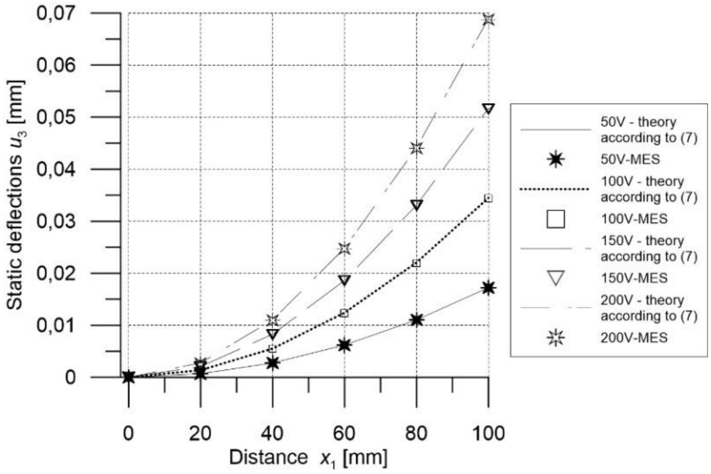


Fig. 5. Comparison of static deflections obtained with the FEM simulations and analytical solutions from equation (7) [source: own study]

6. MODAL ANALYSIS

In order to determine the d_{311} effect on dynamic behavior of the activator a numerical modal analyzes were performed. In the FEM simulations the Lanczos method was used [[11],[13]] in order to determine the natural frequencies and mode shapes of free vibrations for three load cases. The first one was an open system, where on the upper and the bottom surface had no applied voltage potential (Case 1). The next was a closed system, where on both surfaces of the PVDF beam the applied potential equaled 0V (Case 2). The third case was a system made of material with identical mechanical properties as the beam made of PVDF, but with deactivated piezoelectric properties. Modal analysis' results are shown in table 3, where in the case No. 3 the natural frequencies were determined from the formula (8) [[14]]. The determined mode shapes are shown in Figures 6, 7.

$$f_i = \frac{1}{2\pi}\left(\frac{\lambda_i}{a}\right)^2 \sqrt{\frac{EJ}{\rho A}} \qquad (8)$$

where : f_{i} – natural frequencies of free vibrations [Hz],

λ_i – constants: λ_1=1,875, λ_2=4,694, $\lambda_n = \pi n/(2n\text{-}1)$ dla n>2 [[14]],

ρ – mass density of the material,

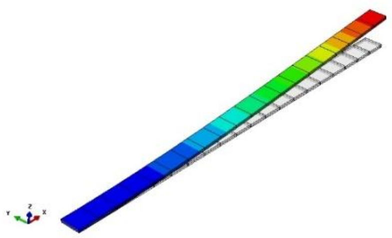
A – the cross-sectional area of the beam,

EJ – bending stiffness: for the compliant direction $E_{11}J_{x1}$, for the rigid direction $E_{22}J_{x2}$.

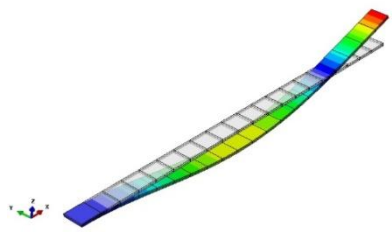
Tab. 3. Natural frequencies of the PVDF beam [Hz]

Mode	Case 1	Case 2	Case 3
	Direction susceptible (plane: x_3-x_1)		
1	17.21	17.14	17.12
2	107.79	107.40	107.26
3	301.64	300.54	300.17
	Direction rigid (plane: x_2-x_1)		
4	85.92	-	85.50
5	533.81	-	531.23

a) Mode 1



b) Mode 2



c) Mode 3

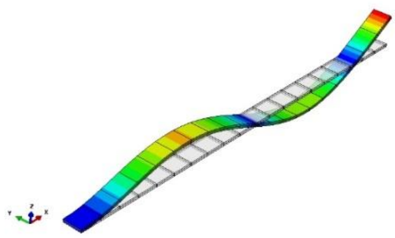
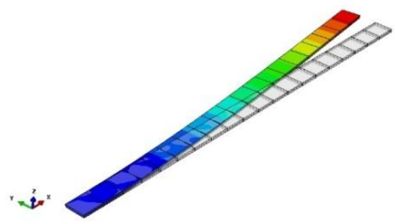


Fig. 6. The bending natural mode shapes of the beam in the compliant direction (plane: x_3-x_1)

a) Mode 4



b) Mode 5

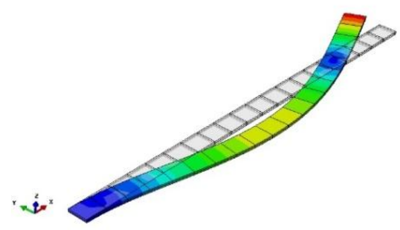


Fig. 7. The bending natural mode shapes of the beam in the rigid direction (plane: x_2-x_1)

Comparison of the results showed that for the compact system (case 2) and the system without of piezoelectric properties (case 3) the increase in natural frequencies didn't exceed 0.15%. It can be assumed that the results are identical. In the case of the open system (case 1) the increase in natural frequencies compared to the system without piezoelectric properties (case 3) didn't exceed the 0.5%. Again, the influence of the piezoelectric effect isn't important.

7. CONCLUSION

This article deals with the influence of piezoelectric effect on static and dynamic behavior of like bimorph – beam. Numerical model FEM was built in the commercial system Abaqus. In the case of static research on deflection of the beam a good compatibility of the numerical simulations with the analytical model and results published by other authors was obtained. Numerical analyzes' results differed by about 0.2% as compared with the results of analytical calculations, but in comparison to the research presented in the literature the differences don't exceed 0,5%. Thus, in all the analyzed cases the adequacy of the FEM model was proved. In the next step modal analysis of the activator to examine the influence of the piezoelectric effect on the behavior of the system. It was found that the natural frequency of the compact system and the system without the piezoelectric properties are identical. In case of the open system the increase in the natural frequencies in relation to the system without the piezoelectric properties was irrelevant and didn't exceed 5%.

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